EC202A1 – Intermediate Macroeconomic Analysis
Spring 2011, Boston University

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Final Exam

Saturday, May 14, 2011

This is a 120-minute test. There is a total of 120 points allocated across five questions. Use the number of points allocated to each part as a suggestion for how long to spend on that part. I recommend that you attempt all parts before using more time than is suggested for any one part. If you complete some parts in less than the suggested time, use your extra time to revisit parts you may have had trouble with the first time through and to check your work.

Please read the questions carefully and write your answers in the space provided. You can use the backs of the sheets for scrap paper, but to get full credit you must show all relevant work in the space provided.

Please follow my instructions at all times.

Concentrate and think carefully, but try to relax too!

University ID: Solutions

(Please do not include your name.)

1. [30 points total, 3 parts] Consider the simple Solow growth model with no technological progress and no employment growth. The economy is described by the production function $Y = A\sqrt{K}\sqrt{N}$. Technology, A, and the number of workers, N, are constant. The capital stock, K, depreciates at the fixed rate δ per period, and the economy saves a fixed proportion, s, of output, Y, per period. Assume throughout that taxes and government expenditure are zero.

a) [10 points] Derive the expression for the steady state capital-labor ratio. Show all of your work.

answer:

<u>production function</u> (1)

$$Y = A\sqrt{K}\sqrt{N}$$

$$\frac{Y}{N} = \frac{A\sqrt{K}\sqrt{N}}{N}$$

$$\frac{Y}{N} = \frac{A\sqrt{K}}{\sqrt{N}} = A\sqrt{\frac{K}{N}}.$$

<u>investment</u> (2)

saving = sY

and, by equilibrium in the goods market,

investment = saving

$I_t = sY_t$.

<u>capital accumulation</u>

$$K_{t+1} = K_t - \delta K_t + I_t$$

$$\frac{K_{t+1}}{N} = \frac{K_t}{N} - \delta\frac{K_t}{N} + s\frac{Y_t}{N} \qquad \text{[substituted (2)]}$$

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = sA\sqrt{\frac{K_t}{N}} - \delta\frac{K_t}{N}. \qquad \text{[substituted (1)]}$$

<u>steady state</u>

In the steady state, the capital stock will be constant at some level K^*:

$$\frac{K^*}{N} - \frac{K^*}{N} = sA\sqrt{\frac{K^*}{N}} - \delta\frac{K^*}{N}$$

$$sA\sqrt{\frac{K^*}{N}} = \delta\frac{K^*}{N}$$

$$\frac{sA}{\delta} = \sqrt{\frac{K^*}{N}} \quad \rightarrow \quad \frac{K^*}{N} = \left(\frac{sA}{\delta}\right)^2.$$

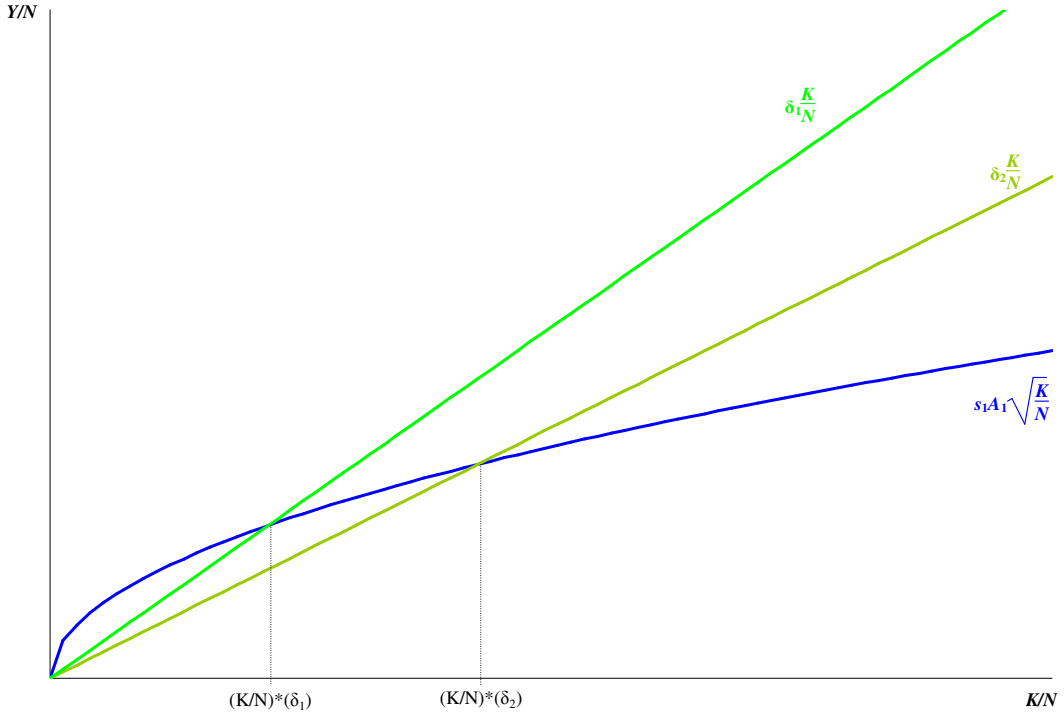
b) [10 points] Suppose that the depreciation rate decreases to $\delta' < \delta$ but that technology, the saving rate and employment stay constant.

i. Show the initial steady state and the new steady state in a carefully-labeled graph. You do not need to include the production function.

ii. Describe briefly how the economy will converge to the new steady state, starting from the old steady state.

answer:

i.



(Note that the graph uses δ_1 for δ and δ_2 for δ'. So $\delta_2 < \delta_1$.)

ii. Initially, investment per worker equals depreciation per worker. In the period in which the depreciation rate decreases, depreciation per worker will immediately decrease, but the capital-labor ratio will not change immediately. The capital-labor ratio will be higher in the next period due to the fact that investment is now outpacing depreciation though, leading to a higher level of output per worker. This in turn will lead to higher investment per worker, a yet higher capital-labor ratio and so on. The net change in the capital-labor ratio will decrease in successive periods – because investment per worker increases at a decreasing rate as the capital-labor ratio increases due to the concavity of the production function, while depreciation per worker increases linearly – and eventually become zero. At this point, the economy will have converged to the new steady state, and the capital-labor ratio will once again be constant, now at its higher steady state level.

c) [10 points] Assume that the depreciation rate is 8%, the saving rate is 40%, and the level of technology is 15.

i. Calculate the steady state levels of output per worker, investment per worker, and consumption per worker.

ii. If output per worker is otherwise known as labor productivity, how would you define capital productivity? Calculate the level of capital productivity in the steady state.

answer:

i.

$$\frac{Y}{N} = A\sqrt{\frac{K}{N}}, \text{ so } \frac{Y^*}{N} = A\sqrt{\frac{K^*}{N}} = A\left(\frac{sA}{\delta}\right) = A^2\left(\frac{s}{\delta}\right) \rightarrow \frac{Y^*}{N} = (15)^2(0.4/0.08) = 1125.$$

$$I_t = sY_t \rightarrow \frac{I^*}{N} = s\frac{Y^*}{N} = 0.4(1125) = 450.$$

$C = Y - I$ [from equilibrium in the goods market, and $G = 0$, as given]

$C = Y - sY$

$$\frac{C^*}{N} = (1 - s)\frac{Y^*}{N} = (1 - 0.4)(1125) = 675.$$

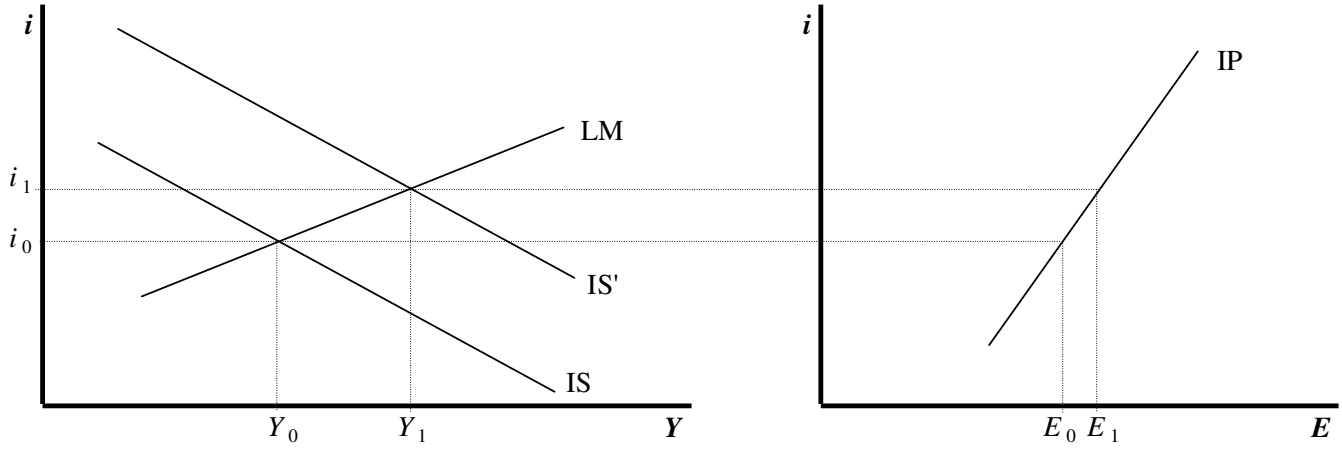
ii. Capital productivity is defined as output per unit of capital.

$$\left(\frac{Y}{K}\right)^* = \frac{Y^*}{N} / \frac{K^*}{N} = A^2\left(\frac{s}{\delta}\right) / \left(\frac{sA}{\delta}\right)^2 = 1 / \left(\frac{s}{\delta}\right) = 0.08/0.4 = 0.2.$$

2. [15 points total, 2 parts] Consider the general open-economy *IS-LM* model. The economy is currently in an initial short-run equilibrium.

a) [7 points] Suppose that the domestic government decides to cut taxes. Illustrate this situation graphically, carefully labeling all axes and curves. Also label the values of output, the interest rate, and the exchange rate corresponding to the initial equilibrium and the new equilibrium.

answer:



b) [8 points] State how each of the following variables will have changed in the new equilibrium relative to the initial equilibrium, and explain briefly why.

i. Consumption.

ii. Investment.

iii. Net exports.

iv. The government budget balance.

answer:

i. Consumption will increase because taxes decrease and output increases, both increasing disposable income, upon which consumption depends positively in the model.

ii. The change in investment is ambiguous. Investment depends positively on output and negatively on the interest rate in the model. Both have increased, exerting opposite pressures on investment.

iii. Net exports decrease. The nominal exchange rate (and by definition the real exchange rate, since prices are held constant in the short run) appreciates after the tax cut. By assumption that the Marshall-Lerner condition holds, this alone will decrease net exports. (Exports will decrease because they become more expensive to other countries; imports will increase because they become cheaper for us, while their value decreases, though this latter effect is assumed to be relatively small.) In addition, the higher equilibrium output means that there will be an additional factor increasing imports. Increased imports and decreased exports both serve to decrease net exports.

iv. The government budget balance is just public saving, or $T - G$. Taxes have fallen (by construction of the problem) and government expenditure has remained constant (by implicit assumption, since it is an exogenous variable and the problem does not state that it has changed). Therefore, the budget balance has decreased (deteriorated, worsened etc.).

3. [30 points total, 4 parts] Consider an economy characterized by the following Aggregate Supply and Aggregate Demand relations:

$P = P^e(0.5 + 0.0005Y)$ [*AS*]
$P = 10 - 0.008Y + 0.02c_0$ [*AD*].

a) [5 points]

i. Confirm that the natural level of output is 1000.

ii. If the economy starts in medium run equilibrium with consumer confidence of $c_0 = 150$, find: the level of output; the actual price level; and the expected price level.

answer:

i. When the economy is in its natural state, the expected price level is equal to the actual price level. Substitute this condition into the *AS* curve:

$P = P^e(0.5 + 0.0005Y)$
$P = P(0.5 + 0.0005Y)$
$1 = 0.5 + 0.0005Y$
$0.0005Y = 1 - 0.5$
$Y_n = 0.5/0.0005 = 1000.$

ii. In medium run equilibrium, output will be equal to the natural level, and the price level (which will, by definition, be equal to the expected price level) will be determined by the height of the *AD* curve:

$P = 10 - 0.008Y + 0.02c_0$
$P = 10 - 0.008(1000) + 0.02(150) = 10 - 8 + 3 = 5 = P^e.$

b) [10 points] Now suppose that the economy suffers a permanent decrease in consumer confidence, to $c_0 = 100$, i.e. consumer confidence remains at this new level indefinitely.

i. Find the short run equilibrium level of output and the price level that would arise if there were no adjustment to the new output level in the labor market.

ii. Find the short run equilibrium level of output and the price level accounting for adjustment in the labor market to the change in output. (You can round your answers to one or two decimal places, but you should use up to five decimal places in your intermediate calculations where necessary.)

answer:

i. If there were no adjustment in the labor market, the price level would stay fixed at the level found above – namely 5 – and output would be determined by the aggregate demand curve alone at this price level:

$P = 10 - 0.008Y + 0.02c_0$
$5 = 10 - 0.008Y + 0.02(100)$
$0.008Y = 10 + 2 - 5$
$\hat{Y} = 7/0.008 = 875.$

ii. With adjustment in the labor market as well as the goods market and financial markets, the short run equilibrium will be determined by the intersection of the aggregate supply and aggregate demand curves:

$AS = AD$
$P^e(0.5 + 0.0005Y) = 10 - 0.008Y + 0.02c_0$
$5(0.5 + 0.0005Y) = 10 - 0.008Y + 0.02(100)$ [because price expectations stay fixed in the short run]
$2.5 + 0.0025Y = 10 - 0.008Y + 2$
$0.0105Y = 10 + 2 – 2.5$
$Y' = 9.5/0.0105 = 904.76.$

$P' = 10 - 0.008Y' + 0.02c_0$
$P' = 10 - 0.008(904.76) + 0.02(100) = 10 – 7.24 + 2 = 4.76.$

c) [5 points] Starting from the short run equilibrium in which the labor market has adjusted to the new output level, and continuing to assume that $c_0 = 100$, now suppose that price expectations adjust, such that P^e becomes equal to the actual price level you found for this short run equilibrium.

i. Find the subsequent short run equilibrium output and price level. (You can round your answers to one or two decimal places, but you should use up to five decimal places in your intermediate calculations where necessary.)

answer:

$AS = AD$
$P^e(0.5 + 0.0005Y) = 10 - 0.008Y + 0.02c_0$
$4.76(0.5 + 0.0005Y) = 10 - 0.008Y + 0.02(100)$
$2.38095 + 0.00238Y = 10 - 0.008Y + 2$
$0.01038Y = 10 + 2 - 2.38095$
$Y'' = 9.61905/0.01038 = 926.69.$

$P'' = 10 - 0.008Y'' + 0.02c_0$
$P'' = 10 - 0.008(926.69) + 0.02(100) = 10 – 7.41353 + 2 = 4.59.$

d) [10 points] Continue to assume that $c_0 = 100$. Suppose that the economy has completed its convergence to the new medium run equilibrium.

i. Find the medium run equilibrium: output level; price level; and expected price level.

ii. State how consumption and investment will have changed in the new medium run equilibrium compared to the initial medium run equilibrium. Explain.

answer:

i. In medium run equilibrium, output will be equal to the natural level (which hasn't changed, so is still 1000), and the price level (which will, by definition, be equal to the expected price level) will be determined by the height of the *AD* curve:

$P = 10 - 0.008Y + 0.02c_0$
$P = 10 - 0.008(1000) + 0.02(100) = 10 - 8 + 2 = 4 = P^e$.

ii. Investment will be unambiguously higher. Our usual assumptions about investment are that it increases with output and decreases with the interest rate. Output has not changed. To discern what has happened to the interest rate, we need to think about the *IS-LM* diagram. The price level has fallen (while the nominal money supply has stayed constant by implicit assumption), so the real value of the money supply is higher, and the *LM* curve must therefore have shifted downwards. The drop in consumer confidence pushed the *IS* curve leftwards. These are the only forces affecting either curve, and both serve to push the interest rate lower; we thus know that the interest rate has decreased. Constant output and a lower interest rate push investment up, so we therefore know with certainty that investment will have increased.

Consumption will be unambiguously lower. Autonomous consumption has fallen by construction of the problem. The part of consumption depending on disposable income has not changed because output returns to its natural level (and taxes are constant by implicit assumption). Put together, this means that the net change in consumption is just the drop in autonomous consumption – namely, 50, from 150 to 100.

We can in fact state by exactly how much each has changed. Since $Y = C + I + G$ in goods market equilibrium, and because output does not change from its natural level in one medium run equilibrium compared to the other (while government expenditure is constant by implicit assumption), investment must have increased by exactly as much as consumption has decreased. It must therefore be the case that consumption is lower by 50 in the new medium run equilibrium while investment is higher by 50.

4. [15 points total, 2 parts] Consider the following behavioral equation and information describing the goods market:

$C = c_0 + c_1 Y_D$

Government expenditure is represented by the exogenous variable G as usual, and investment is represented by the exogenous constant I throughout.

a) [5 points] Find the expression for the equilibrium level of output in this closed-economy goods market.

answer:

$Y = Z$ (for equilibrium in the goods market)
$Y = C + I + G$
$Y = (c_0 + c_1 Y_D) + I + G$
$Y = c_0 + c_1(Y - T) + I + G$
$Y(1 - c_1) = c_0 - c_1 T + I + G$
$Y^{eq} = [1/(1 - c_1)][c_0 - c_1 T + I + G]$.

b) [10 points] Now suppose that the goods market becomes open to trade. The behavioral equation and information of the previous part remain valid, and are augmented by the following:

$IM = m_0 + m_1 Y$
$X = x_1 Y^*$.

Assume that the real exchange rate is fixed at a value of 1 and treat foreign income, Y^*, as fixed.

i. Find the expression for the equilibrium level of output in this open-economy goods market.

ii. Suppose that government expenditure increases by 100. By how much would output have increased in the closed-economy case? By how much will output increase in the open-economy case. Is the increase in output larger in the open-economy or the closed-economy case? Explain briefly and intuitively why.

answer:

i.

$Y = C + I + G + X - IM$
$Y = c_0 + c_1(Y - T) + I + G + x_1 Y^* - (m_0 + m_1 Y)$
$Y^{eq} = [1/(1 - c_1 + m_1)][c_0 - c_1 T + I + G + x_1 Y^* - m_0]$.

ii. In response to a one-unit change in government expenditure, equilibrium output will change by the multiplier. In the open-economy case, the increase in equilibrium output from a 100-unit increase in government expenditure will therefore be $100/(1 - c_1 + m_1)$. In the closed-economy case, it would have been $100/(1 - c_1)$.

The multiplier – and therefore the increase in output following the increase in government expenditure – is clearly larger in the closed-economy case. Mechanically, the addition of the positive propensity-to-import parameter makes the denominator larger and therefore the ratio smaller in the open-economy case. Intuitively, the availability of imports means that we will spend less of each additional dollar on domestically-produced goods because we will then be spending part of each additional dollar on foreign-produced goods.

When government expenditure increases in the open-economy case, demand first increases directly, then the corresponding increase in output has two indirect effects through the multiplier process: consumption increases, meaning that domestic demand for total goods is higher; but imports also increase, meaning that demand for domestically-produced goods has not increased by as much. Only the additional demand for domestically-produced goods will contribute to a further increase in domestic income. If there were no foreign-produced goods available (i.e. in a closed economy), the entire increase in consumption would have gone to domestically-produced goods.

5. [30 points total, 3 parts] Consider the closed-economy *IS-LM* model. The following behavioral equations and exogenous variables describe the economy:

$C = 200 + 0.4Y_D$
$I = 200 + 0.3Y - 1200i$
$(M/P)^d = 1.5Y - 4000i$

$G = 225$
$T = 100$
$(M/P)^s = 2425.$

a) [10 points] Derive the *IS* relation. Derive the *LM* relation. Find the short-run equilibrium output level and interest rate.

answer:

goods market

$Y = Z$
$Y = C + I + G$
$Y = (200 + 0.4(Y - 100)) + (200 + 0.3Y - 1200i) + 225$
$Y - 0.4Y - 0.3Y = 200 - 0.4(100) + 200 - 1200i + 225$
$0.3Y = 585 - 1200i$
$Y = 1950 - 4000i$ (*IS* curve)

financial market

$(M/P)^s = (M/P)^d$
$2425 = 1.5Y - 4000i$
$4000i = 1.5Y - 2425$
$i = 0.000375Y - 0.60625$ (*LM* curve)

equilibrium

Substitute *LM* into *IS*:

$Y = 1950 - (1.5Y - 2425)$
$Y + 1.5Y = 1950 + 2425$
$2.5Y = 4375$
$Y^* = 4375/2.5 = 1750.$

Substitute Y^* into *LM*:

$i = 0.000375(1750) - 0.60625$
$i^* = 0.65625 - 0.60625 = 0.05 = 5\%.$

b) [10 points] Now suppose that the central bank reduces the real money supply to $(M/P)^s = 1825$.

i. Find the new short-run equilibrium output level and interest rate.

ii. Calculate the level of private saving before and after the reduction in the real money supply.

iii. Without calculating investment, state by how much it must change following the monetary contraction. Explain.

answer:

i. Derive the *LM* curve again with the new level of the real money supply:

$(M/P)^s = (M/P)^d$
$1825 = 1.5Y - 4000i$
$4000i = 1.5Y - 1825$
$i = 0.000375Y - 0.45625$ (new *LM* curve).

Now substitute this new *LM* into the unchanged *IS*:

$Y = 1950 - (1.5Y - 1825)$
$2.5Y = 3775$
$Y^{**} = 3775/2.5 = 1510;$

and Y^{**} back into the new *LM*:

$i = 0.000375(1510) - 0.45625$
$i^{**} = 0.56625 - 0.45625 = 0.11 = 11\%.$

ii. Private saving is given by $S = Y_D - C$.

$S^* = (Y^* - T) - C^*$
$= 1750 - 100 - (200 + 0.4(1750 - 100))$
$= 0.6(1650) - 200$
$= 790.$

$S^{**} = 0.6(1410) - 200$
$= 646.$

iii. Equilibrium in the goods market implies that investment is equal to total saving. Total saving is equal to public saving plus private saving. Public saving has not changed (by implicit assumption that taxes and government expenditure are being held fixed at their stated levels). Therefore, the change in investment must be equal to the change in private saving. From what we just calculated, it can be seen that private saving falls by 144. Investment will hence fall by 144 as well, which can be verified by plugging equilibrium values of output and the interest rate into the investment function. (A monetary contraction will always lead to a decline in investment, because it will lead to lower equilibrium output and a higher equilibrium interest rate, both of which serve to decrease investment. This is exactly what we have seen in this case.)

c) [10 points] Now suppose that, with the real money supply still at $(M/P)^s = 1825$, the government decides that it would like the output level to be 1750, and that it will attempt to achieve this by changing the level of government expenditure, G, only.

i. Find the new level of G that will achieve this output target.

ii. What is the value of the multiplier in this economy?

answer:

i. We want to increase government expenditure such that the *IS* curve shifts right by just enough to achieve the initial output level. The equilibrium point will travel up along the new *LM* curve as the *IS* curve shifts. So, one way to proceed is to find the point on the new *LM* curve where we want to eventually end up, then plug the associated values for the interest rate and output into the *IS* curve, and back out the required level of government spending. There are other ways to proceed, but this is probably the simplest.

$i = 0.000375Y - 0.45625$
$= 0.000375(1750) - 0.45625$
$= 0.65625 - 0.45625$
$= 0.2 = 20\%$.

$Y = C + I + G$
$Y = (200 + 0.4(Y - 100)) + (200 + 0.3Y - 1200i) + G$
$1750 = 200 + 0.4(1750) - 40 + 200 + 0.3(1750) - 1200(0.2) + G$
$G = 0.3(1750) + 40 - 400 + 240$
$= 525 + 280 - 400$
$= 405$.

ii. The multiplier when we consider the goods and financial markets simultaneously is not the same as the multiplier when we consider the goods market in isolation. So we can't just use the expressions from the previous problem. However, we can apply the same simple logic from the previous problem. There, the change in output was calculated as the change in government expenditure times the multiplier. If we just reverse that calculation, the multiplier must be given by the change in output divided by the change in government expenditure. Here, the goal is to increase output from 1510 to 1750, and we have just calculated that this can be accomplished by increasing government expenditure from 225 to 405. Therefore,

multiplier $= \Delta Y/\Delta G = (1750 - 1510)/(405 - 225) = 240/180 = 4/3 = 1.333\ldots$

A much more tedious way to proceed would have been to derive a symbolic expression for equilibrium output (or try to remember it from class and practice problems), and calculate the multiplier by substituting in the particular parameters from this problem. This method can be used to verify that the calculation above is correct.