EC202C1 – Intermediate Macroeconomic Analysis
Spring 2012, Boston University

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Final Exam

Tuesday, May 8, 2012

This is a 120-minute exam. There is a total of 120 points allocated across four questions. Use the number of points allocated to each part as a suggestion for how long to spend on that part. I recommend that you attempt all parts before using more time than is suggested for any one part. If you complete some parts in less than the suggested time, use your extra time to revisit parts you may have had trouble with the first time through and to check your work.

Please read the questions carefully and write your answers in the space provided. You can use the backs of the sheets for scrap paper, but to get full credit you must show all relevant work in the space provided.

Please follow my instructions at all times.

Concentrate and think carefully, but try to relax too!

University ID: Solutions

(Please do not include your name.)

1. [34 points total, 4 parts] Consider the closed-economy *IS-LM* model. The following behavioral equations and exogenous variables describe the economy:

$C = 440 + 0.2Y_D$
$I = 240 + 0.4Y - 2000i$
$(M/P)^d = 2Y - 4000i$

$G = 800$
$T = 400$
$(M/P)^s = 5558.$

a) [12 points] Derive the *IS* relation. Derive the *LM* relation. Verify that the short-run equilibrium output level is 2985. Find the short-run equilibrium interest rate.

answer:

goods market

$Y = Z$
$Y = C + I + G$
$Y = (440 + 0.2(Y - 400)) + (240 + 0.4Y - 2000i) + 800$
$Y - 0.2Y - 0.4Y = 440 - 0.2(400) + 240 - 2000i + 800$
$0.4Y = 1400 - 2000i$
$Y = 3500 - 5000i$ (*IS* curve)

financial market

$(M/P)^s = (M/P)^d$
$5558 = 2Y - 4000i$
$4000i = 2Y - 5558$
$i = 0.0005Y - 1.3895$ (*LM* curve)

equilibrium

Substitute *LM* into *IS*:

$Y = 3500 - 5000(0.0005Y - 1.3895)$
$Y + 2.5Y = 3500 + 6947.5$
$3.5Y = 10447.5$
$Y^* = 10447.5/3.5 = 2985.$

Substitute Y^* into *LM*:

$i = 0.0005(2985) - 1.3895$
$i^* = 1.4925 - 1.3895 = 0.103 = 10.3\%.$

b) [8 points] Now suppose that there is a drop in consumer confidence. Specifically, suppose that the autonomous component of Consumption falls from 440 to 300. Find the new short-run equilibrium output level and interest rate.

answer:

$Y = Z$
$Y = C + I + G$
$Y = (300 + 0.2(Y - 400)) + (240 + 0.4Y - 2000i) + 800$
$Y - 0.2Y - 0.4Y = 300 - 0.2(400) + 240 - 2000i + 800$
$0.4Y = 1260 - 2000i$
$Y = 3150 - 5000i$ (IS')

$Y = 3150 - 5000(0.0005Y - 1.3895)$ (same LM)
$3.5Y = 10097.5$
$Y^{**} = 2885.$

$i = 0.0005(2885) - 1.3895$
$i^{**} = 1.4425 - 1.3895 = 0.053 = 5.3\%.$

c) [8 points] Now suppose that, with consumer confidence still at 300, the central bank decides that it would like output to return to the level it was at in the original equilibrium, from part a). Find the new level of the real money supply, $(M/P)^s$, that will achieve this output target.

answer:

To boost output from 2885 back to 2985, the central bank will have to increase the real money supply, which will shift the *LM* curve downward. When it does so, adjustments in financial markets will put downward pressure on the interest rate, which will cause adjustments in the goods market, and so on as the economy travels down the unchanged *IS*′ curve to its new equilibrium point. We know what level of output will be obtained in this equilibrium by construction (2985), and we can use the height of the unchanged *IS*′ curve to determine what the corresponding interest rate must be in order for both markets to be in simultaneous equilibrium.

$Y = 3150 - 5000i$
$2985 = 3150 - 5000i$
$5000i = 3150 - 2985$
$i^{***} = 165/5000 = 0.033 = 3.3\%$.

Now, we can find the demand for real balances that will arise at this equilibrium output level and interest rate pair. And since the demand for real balances must equal the real money supply for financial markets to be in equilibrium, this will give us the required real money supply to support the desired output level (2985) as an outcome of simultaneous equilibrium in goods and financial markets.

$(M/P)^s = (M/P)^d$
$(M/P)^s = 2Y - 4000i$
$(M/P)^s = 2(2985) - 4000(0.033)$
$(M/P)^s = 5838$.

Some people plugged the interest rate from the previous part along with the desired output level into the money demand function. This is incorrect because it does not account for adjustments in the goods market caused by adjustments in financial markets. In other words, it identifies a point of equilibrium in financial markets that does not correspond to equilibrium in the goods market.

d) [6 points] Verify that total (national) saving is equal to Investment when consumer confidence is still at 300 and the real money supply is at the new level that you found in the previous part. (You can explain this in words and/or symbols if you were unable to complete previous parts.)

answer:

private saving = Y_D - C
= (2985 - 400) - (300 + 0.2(2985 - 400))
= 2585 - 300 - 0.2(2585)
= 0.8(2585) - 300
= 1768

public saving = T - G
= 400 - 800
= -400

Total saving = private saving + public saving
= 1768 - 400
= 1368.

I = 240 + 0.4Y - 2000i
= 240 + 0.4(2985) - 2000(0.033)
= 240 + 1194 - 66
= 1368.

So, saving equals investment in the equilibrium corresponding to the drop in consumer confidence and the monetary expansion (Y*** = 2985, i*** = 3.3%), as required (and as implied by equilibrium in the goods market).

2. [30 points total, 3 parts] Consider the full Solow growth model with exogenous technological progress and employment growth. The economy is described by the production function $Y = \sqrt{K}\sqrt{AN}$. Technology, A, grows at the fixed percentage rate g_A per period, and employment, N, grows at the fixed percentage rate g_N per period. The capital stock, K, depreciates at the fixed rate δ per period, and the economy saves a fixed proportion, s, of output, Y, per period. Assume throughout that taxes and government expenditure are zero. You can assume that "necessary investment" is given by $(\delta + g_A + g_N)K$ without any derivation or explanation.

a) [12 points] Derive the expression for the steady state level of capital per effective worker. Show all of your work.

answer:

First, find the intensive form of the production function:

$$\frac{Y}{AN} = \frac{\sqrt{K}\sqrt{AN}}{AN} = \sqrt{\frac{K}{AN}}.$$

In the steady state, capital per effective worker will be constant, which requires that actual investment must equal "necessary investment":

$I_t = (\delta + g_A + g_N)K_t$

$\frac{I_t}{A_tN_t} = (\delta + g_A + g_N)\frac{K_t}{A_tN_t}$ [divided by effective workers]

$\frac{sY_t}{A_tN_t} = (\delta + g_A + g_N)\frac{K_t}{A_tN_t}$ [because saving equals investment in equilibrium, and saving is s% of output]

$s\sqrt{\frac{K_t}{A_tN_t}} = (\delta + g_A + g_N)\frac{K_t}{A_tN_t}$ [inserting the production function]

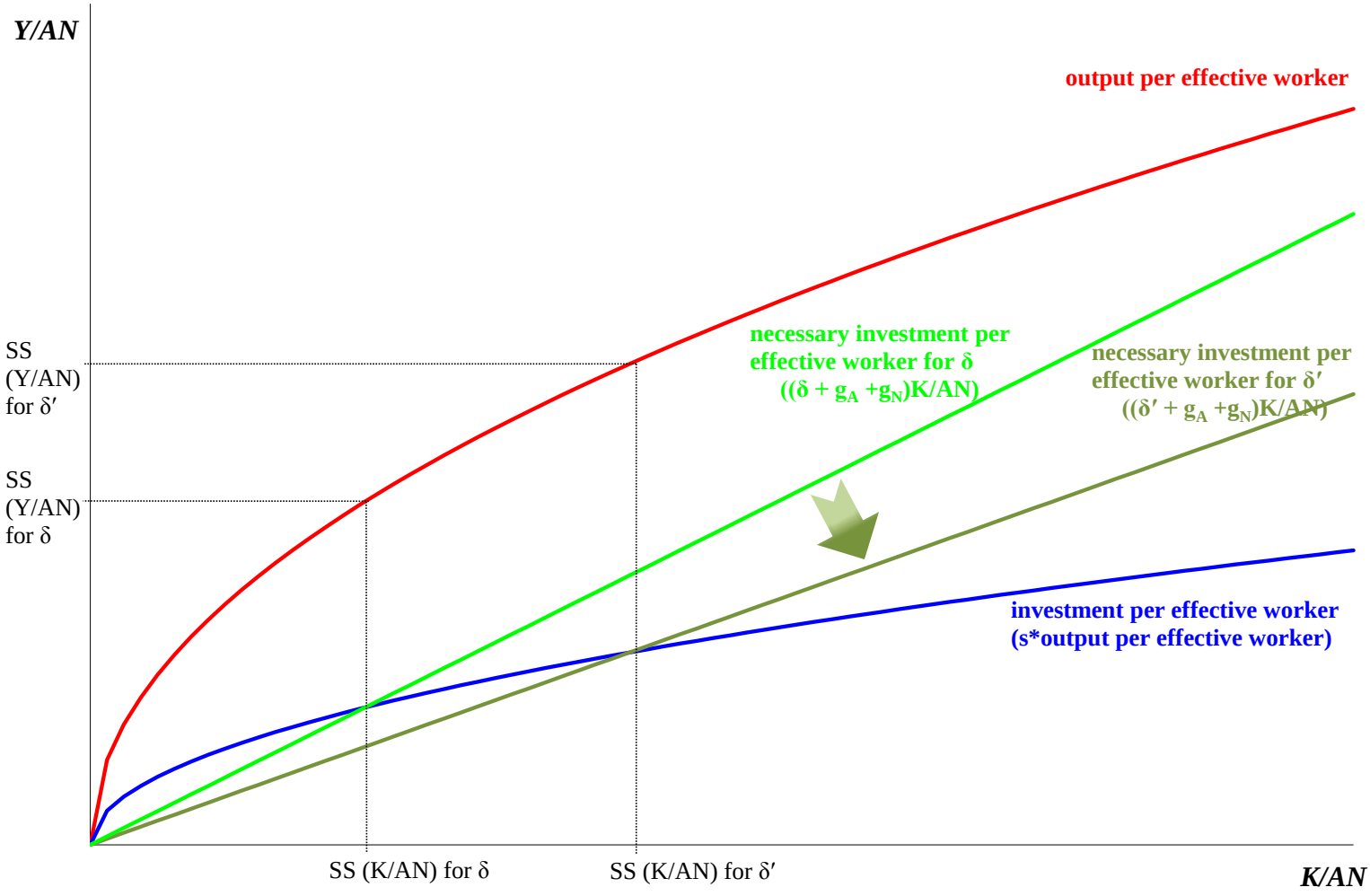
$s\sqrt{\left(\frac{K}{AN}\right)^*} = (\delta + g_A + g_N)\left(\frac{K}{AN}\right)^*$ [because K/AN is constant in the SS]

$s/(\delta + g_A + g_N) = \sqrt{\left(\frac{K}{AN}\right)^*}$

$\left(\frac{K}{AN}\right)^* = \left(\frac{s}{(\delta + g_A + g_N)}\right)^2.$

b) [10 points] Suppose that the depreciation rate decreases to $\delta' < \delta$ but that the saving rate and the technology and employment growth rates stay constant. Illustrate the initial steady state and the new steady state on a carefully-labeled graph. For each, indicate the steady state level of capital per effective worker and the steady state level of output per effective worker.

answer:



c) [8 points] Suppose that the saving rate is 16%, the depreciation rate is 10%, employment growth is 2%, and technology growth is 4%. The economy has been in its corresponding steady state for several periods. In some period t, you observe that the level of technology is $A_t = 600$. Calculate the level of consumption per worker in period t and in period t+1.

answer:

$$\left(\frac{K}{AN}\right)^* = \left(\frac{s}{(\delta + g_A + g_N)}\right)^2 = \left(\frac{0.16}{(0.1 + 0.02 + 0.04)}\right)^2 = \left(\frac{0.16}{(0.16)}\right)^2 = 1^2 = 1.$$

$$\left(\frac{Y}{AN}\right)^* = \sqrt{\left(\frac{K}{AN}\right)^*} = \sqrt{1} = 1.$$

$Y = C + I$ [equilibrium in the goods market, and $G = 0$ as given above]

$C = Y - sY$ [rearranging, and noting that investment equals saving in equilibrium and saving is the saving rate times output as usual]

$\left(\frac{C}{AN}\right)^* = (1 - s)\left(\frac{Y}{AN}\right)^*$ [simplifying, dividing both sides by AN, and evaluating at the SS]

$$\left(\frac{C}{AN}\right)^* = (1 - 0.16)\times 1 = 0.84.$$

Also note, completely mechanically, that (C/N) = (C/AN)×A. So, in any period t when the economy is on its steady state growth path,

$\left(\frac{C}{N}\right)^*_t = \left(\frac{C}{AN}\right)^* \times A_t = 0.84A_t$, so that, at $A_t = 600$,

$$\left(\frac{C}{N}\right)^*_t = 0.84\times 600 = 504.$$

For period t+1, one can repeat this same process, instead using $A_{t+1} = 600(1.04) = 624$ by the assumption of technology growing at a fixed percentage rate per period, given as 4% in this case, and noting that consumption per effective worker is constant from period to period in the steady state, at 0.84 in this case:

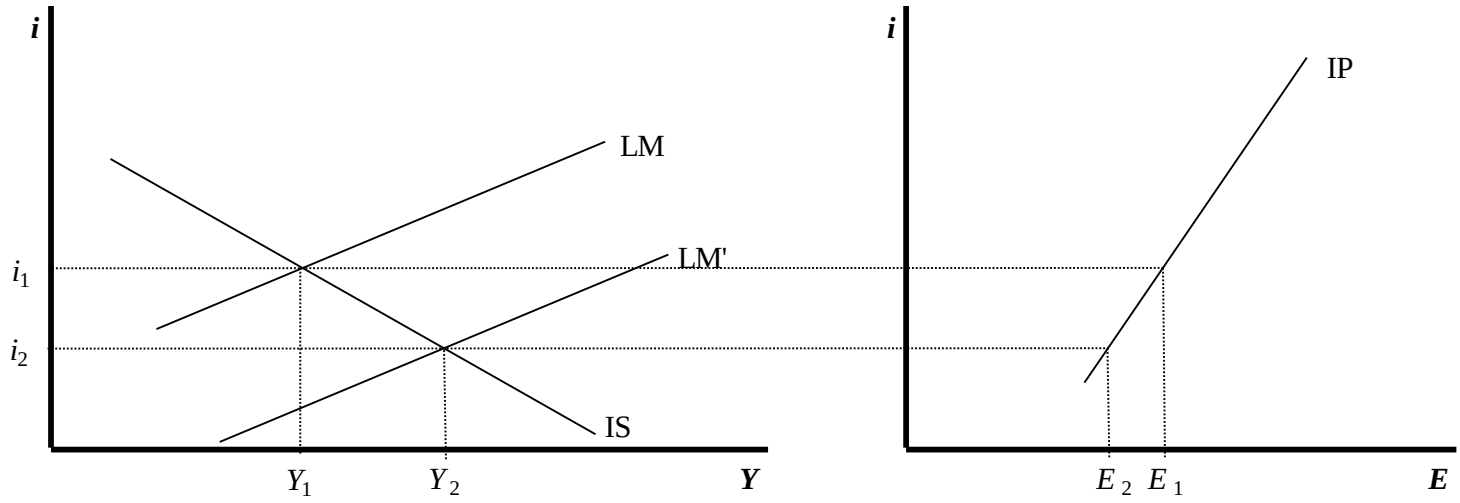
$$\left(\frac{C}{N}\right)^*_{t+1} = 0.84\times 624 = 524.16.$$

Equivalently, one can show using the growth rate rules that consumption per worker grows at the growth rate of technology in the steady state. (Indeed, to confirm, 504×1.04 = 524.16, as above.)

3. [22 points total, 2 parts] Consider the general open-economy *IS-LM* model. The economy is currently in an initial short-run equilibrium.

a) [12 points] Suppose that the domestic central bank decides to increase the real money supply. Illustrate this situation graphically, carefully labeling all axes and curves. Also label the values of output, the interest rate, and the exchange rate corresponding to the initial equilibrium and the new equilibrium.

answer:



A monetary expansion shifts the *LM* curve down. Equilibrium output will increase; the equilibrium interest rate and exchange rate will decrease.

b) [10 points] State how each of the following variables will have changed in the new equilibrium relative to the initial equilibrium, and explain briefly why.

i. Consumption; ii. Investment; iii. Net exports; iv. Private Saving; v. Foreign Output.

answer:

i. Consumption depends positively on Disposable Income. Disposable Income is higher in the new equilibrium because output is higher and taxes have not been mentioned so should be assumed not to have changed. So Consumption will be higher in the new equilibrium.

ii. Investment depends positively on output and negatively on the interest rate. So, in the new equilibrium, the higher output will push Investment up, and the lower interest rate will also push Investment up. Therefore, Investment will be higher in the new equilibrium.

iii. The change in Net Exports is ambiguous. The exchange rate has depreciated, which, by the Marshall-Lerner Condition, pushes Net Exports up, holding output constant. (Domestically-produced goods become cheaper to foreign consumers, so domestic exports increase. Foreign-produced goods become more expensive to domestic consumers, so domestic imports decrease. The value of domestic imports also decreases, but the assumption that the Marshall-Lerner Condition holds implies that this third effect is small relative to the first two.) But output has also increased, pushing imports back up and thus Net Exports back down; and (unlike a case in which the effects of an exogenous depreciation in the goods market in isolation are analyzed holding the interest rate fixed) we have no way of knowing if this countervailing output effect is large enough to fully offset the exchange rate effect.

iv. Private Saving increases. Disposable Income and Consumption each increase, as argued above. Since Private Saving is just the difference between Disposable Income and Consumption, it looks on the face of it like the change in Private Saving is ambiguous. But we know that Consumption increases by less, because a marginal propensity to consume less than one is a basic implicit assumption of the model. That is, for each additional dollar of Disposable Income, only part of that dollar goes towards additional Consumption, with the rest going towards additional Private Saving.

v. Foreign Output is just what we call Y^* in this model, and it is an exogenous constant. So Foreign Output remains constant following the domestic monetary expansion. A lot of people tried to answer this by thinking about the change in domestic Net Exports from the perspective of the foreign country. This is valid enough and received some partial credit if the argument was supported well enough, but it ignores other adjustments that would be going on in all of the foreign markets. What we would really need to do in order to tell the whole story is to fully endogenize output *and* the interest rate in the foreign country. But such an extended model is far too complicated for our purposes, as it requires thinking about one country's shock causing a shock in the other country, and back and forth, on and on. We alluded to this briefly in a classroom discussion, and concluded by saying that we save ourselves a lot of complications without losing too much generality by just retaining the baseline assumption that all of the foreign variables are exogenous from the perspective of the domestic country.

4. [34 points total, 4 parts] Consider an economy characterized by the following Aggregate Supply and Aggregate Demand relations:

$P = P^e(0.5 + 0.0005Y)$ [*AS*]
$P = 15 - 0.008Y - 0.01T$ [*AD*].

a) [5 points]

i. Confirm that the natural level of output is 1000.

ii. If the economy starts in medium run equilibrium with taxes of $T = 300$, find: the level of output; the actual price level; and the expected price level.

answer:

i. When the economy is in its natural state, the expected price level is equal to the actual price level. Substitute this condition into the *AS* curve:

$P = P^e(0.5 + 0.0005Y)$
$P = P(0.5 + 0.0005Y)$
$1 = 0.5 + 0.0005Y$
$0.0005Y = 1 - 0.5$
$Y_n = 0.5/0.0005 = 1000.$

ii. In medium run equilibrium, output will be equal to the natural level, and the price level (which will, by definition, be equal to the expected price level) will be determined by the height of the *AD* curve:

$P = 15 - 0.008Y - 0.01T$
$P_0 = 15 - 0.008(1000) - 0.01(300) = 15 - 8 - 3 = 4 = P^e_0.$

b) [14 points] Now suppose that there is a sharp increase in the price of oil, which causes the Aggregate Supply curve to shift to $P = P^e(0.5875 + 0.0005Y)$ [AS'].

i. Find the short run equilibrium output and price level (accounting for adjustment in all markets, but no immediate adjustment in price expectations) directly caused by the supply shock, assuming that taxes stay constant at $T = 300$.

ii. Find the short run equilibrium output and price level (accounting for adjustment in all markets, but no immediate adjustment in price expectations) that would instead arise if, immediately following the supply shock, the government responded by aggressively cutting taxes to $T = 125$.

answer:

i. Find the intersection of AS' and AD, with price expectations remaining at $P^e_0 = 4$ (i.e. where they were in the initial medium run equilibrium):

$AS' = AD$
$P^e(0.5875 + 0.0005Y) = 15 - 0.008Y - 0.01T$
$4(0.5875 + 0.0005Y) = 15 - 0.008Y - 0.01(300)$
$0.002Y + 0.008Y = 15 - 3 - 2.35$
$0.01Y = 9.65$
$Y' = 9.65/0.01 = 965.$

Plug this into either supply or demand to find the corresponding price level:

$P = 15 - 0.008Y - 0.01T$
$P = 15 - 0.008(965) - 0.01(300)$
$P' = 15 - 7.72 - 3 = 4.28.$

ii. The tax cut causes aggregate demand to shift out to a new AD'. Find the intersection of AS' and AD', with price expectations remaining at $P^e_0 = 4$ (i.e. where they were in the initial medium run equilibrium):

$AS' = AD'$
$P^e(0.5875 + 0.0005Y) = 15 - 0.008Y - 0.01T'$
$4(0.5875 + 0.0005Y) = 15 - 0.008Y - 0.01(125)$
$0.002Y + 0.008Y = 15 - 1.25 - 2.35$
$0.01Y = 11.4$
$Y'' = 11.4/0.01 = 1140.$

Plug this into either supply or demand to find the corresponding price level:

$P = 15 - 0.008Y - 0.01T'$
$P = 15 - 0.008(1140) - 0.01(125)$
$P'' = 15 - 9.12 - 1.25 = 4.63.$

c) [7 points] Continue to assume that the Aggregate Supply curve is $P = P^e(0.5875 + 0.0005Y)$. Also assume that the government has indeed cut taxes to $T = 125$, and will hold them at this new level indefinitely. Start from the corresponding short run equilibrium from the previous part. Now suppose that price expectations adjust, such that P^e becomes equal to the actual price level you found for this short run equilibrium. Find the subsequent new short run equilibrium output and price level. (You can round your answers to one or two decimal places, but you should use six decimal places where necessary in your intermediate calculations.)

answer:

Following the upward supply shock and the rightward demand shift, we find ourselves above the original natural level of output, with a substantially higher price level. But this has all happened within the short run, in which price expectations have been frozen at the original price level of 4. Over the medium run, price expectations will therefore adjust upwards. This will lead to an upward shift of the aggregate supply curve, and to a new short-run equilibrium to the northwest of the previous one. To find this new short-run equilibrium, find the intersection of the aggregate demand curve (which now stays fixed at AD', corresponding to the new, permanent tax level) with the new aggregate supply curve (AS' as before, but now with the new expected price level):

$AS'(P^e_1) = AD'$
$P^e_1(0.5875 + 0.0005Y) = 15 - 0.008Y - 0.01T'$
$4.63(0.5875 + 0.0005Y) = 15 - 0.008Y - 0.01(125)$
$0.002315Y + 0.008Y = 15 - 1.25 - 2.720125$
$0.010315Y = 11.029875$
$Y''' = 11.029875/0.010315 = 1069.3\ldots$

Plug this into either supply or demand to find the corresponding price level:

$P = 15 - 0.008Y - 0.01T'$
$P = 15 - 0.008(1069.3\ldots) - 0.01(125)$
$P''' = 15 - 8.554435 - 1.25 = 5.2\ldots$

Thus, the new actual price level will again be above the new expected price level, leading to further adjustments to price expectations and further corresponding aggregate supply shifts as the economy proceeds through its medium-run convergence.

d) [8 points] Continue to assume that $T = 125$ and that the Aggregate Supply curve is $P = P^e(0.5875 + 0.0005Y)$. Suppose that the economy has completed its convergence to its new medium run equilibrium.

i. Find this new medium run equilibrium: output level; price level; and expected price level.

ii. Comparing this new medium run equilibrium with the initial medium run equilibrium of part a), in which direction has Consumption changed? Explain.

answer:

i. Over the medium run, the economy will converge to its natural level of output. But note that there has been an aggregate supply shock, so that the natural state of the economy has changed. To find the new natural level of output, again impose equality between the actual and expected price level (which is the definition of the natural state), but this time substitute into the new *AS'*:

$P = P^e(0.5875 + 0.0005Y)$
$P = P(0.5875 + 0.0005Y)$
$1 = 0.5875 + 0.0005Y$
$0.0005Y = 1 - 0.5875$
$Y_n' = 0.4125/0.0005 = 825.$

In medium run equilibrium, output will converge to this new natural level, and the price level (which will, again, by definition of the natural state and thus medium run equilibrium, be equal to the expected price level) will be determined by the height of the relevant aggregate demand curve, namely *AD'* (i.e. the price expectation adjustments will cause the aggregate supply curve to shift up over time, and convergence will occur along the new aggregate demand curve corresponding to the tax level of 125, where it has been fixed since the initial drop from 300):

$P = 10 - 0.008Y - 0.01T$
$P_x = 10 - 0.008(825) - 0.01(125) = 15 - 6.6 - 1.25 = 7.15 = P^e_x.$

ii. Consumption does not change! Consumption depends positively on Disposable Income, which is defined as output minus taxes. Between the original and the new medium run equilibria, output has fallen by 175, from 1000 to 825. Meanwhile, taxes have changed by exactly 175 as well, being cut by the government from 300 to 125. The drop in output pushes Disposable Income down by 175, but the tax cut pushes Disposable Income back up by exactly 175. Since Disposable Income does not change on net, neither does Consumption.

(The higher price level is irrelevant as far as the level of Consumption is concerned. As the price level rises, the real value of the money supply – and thus the quantity of real balances held by consumers – deteriorates, pushing the interest rate up. But the quantity of real balances does not determine Consumption. The choice between how much wealth – a stock – to hold as cash instead of bonds is completely distinct from the choice of how much Disposable Income – a flow – to spend on Consumption rather than save – another flow, which is added to the stock of wealth at the end of the "period" – in terms of the implicit micro workings of our model. The

higher interest rate does affect Investment, though, which must mean that Total Saving is likewise affected – but since Consumption does not change, neither does Private Saving, so that the change in Investment in this case must be exactly the same as the change in Public Saving, namely -175, i.e. the size of the tax cut. This can also be seen by looking at the condition for goods market equilibrium: output falls by 175 on the one hand; and on the other hand, Consumption does not change by the argument above while government expenditure is implicitly assumed not to have changed because it has not been mentioned, so that Investment must account for the full fall in output of 175.)