EC202C1 – Intermediate Macroeconomic Analysis
Spring 2012, Boston University

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Second Mid-term Exam (Practice #1)

Thursday, April 5, 2012

This is a 50-minute exam. There is a total of 50 points allocated across two questions. Use the number of points allocated to each part as a suggestion for how long to spend on that part. I recommend that you attempt all parts before using more time than is suggested for any one part. If you complete some parts in less than the suggested time, use your extra time to revisit parts you may have had trouble with the first time through and to check your work.

Please read the questions carefully and write your answers in the space provided. You can use the backs of the sheets for scrap paper, but to get full credit you must show all relevant work in the space provided.

Please follow my instructions at all times.

Concentrate and think carefully, but try to relax too!

Student Number: Solutions

(Please do not include your name.)

1. [35 points total, 4 parts] Consider the simple Solow model with no technological progress and no employment growth. The economy is described by the production function $Y = A\sqrt{K}\sqrt{N}$. Technology, A, and the number of workers, N, are constant. The capital stock, K, depreciates at the fixed rate δ per period, and the economy saves a fixed proportion, s, of output, Y, per period. Assume throughout that taxes and government expenditure are zero.

a) [11 points] Derive the expression for the steady state capital-labor ratio. Show all of your work.

answer:

production function (1)

$$Y = A\sqrt{K}\sqrt{N}$$

$$\frac{Y}{N} = \frac{A\sqrt{K}\sqrt{N}}{N}$$

$$\frac{Y}{N} = \frac{A\sqrt{K}}{\sqrt{N}} = A\sqrt{\frac{K}{N}}.$$

investment (2)

saving = sY

and, by equilibrium in the goods market,

investment = saving

$I_t = sY_t$.

capital accumulation

$$K_{t+1} = K_t - \delta K_t + I_t$$

$$\frac{K_{t+1}}{N} = \frac{K_t}{N} - \delta\frac{K_t}{N} + s\frac{Y_t}{N}$$ [substituted (2) and divided through by N]

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = sA\sqrt{\frac{K_t}{N}} - \delta\frac{K_t}{N}.$$ [substituted (1) and rearranged]

steady state

In the steady state, the capital stock will be constant at some level K^*:

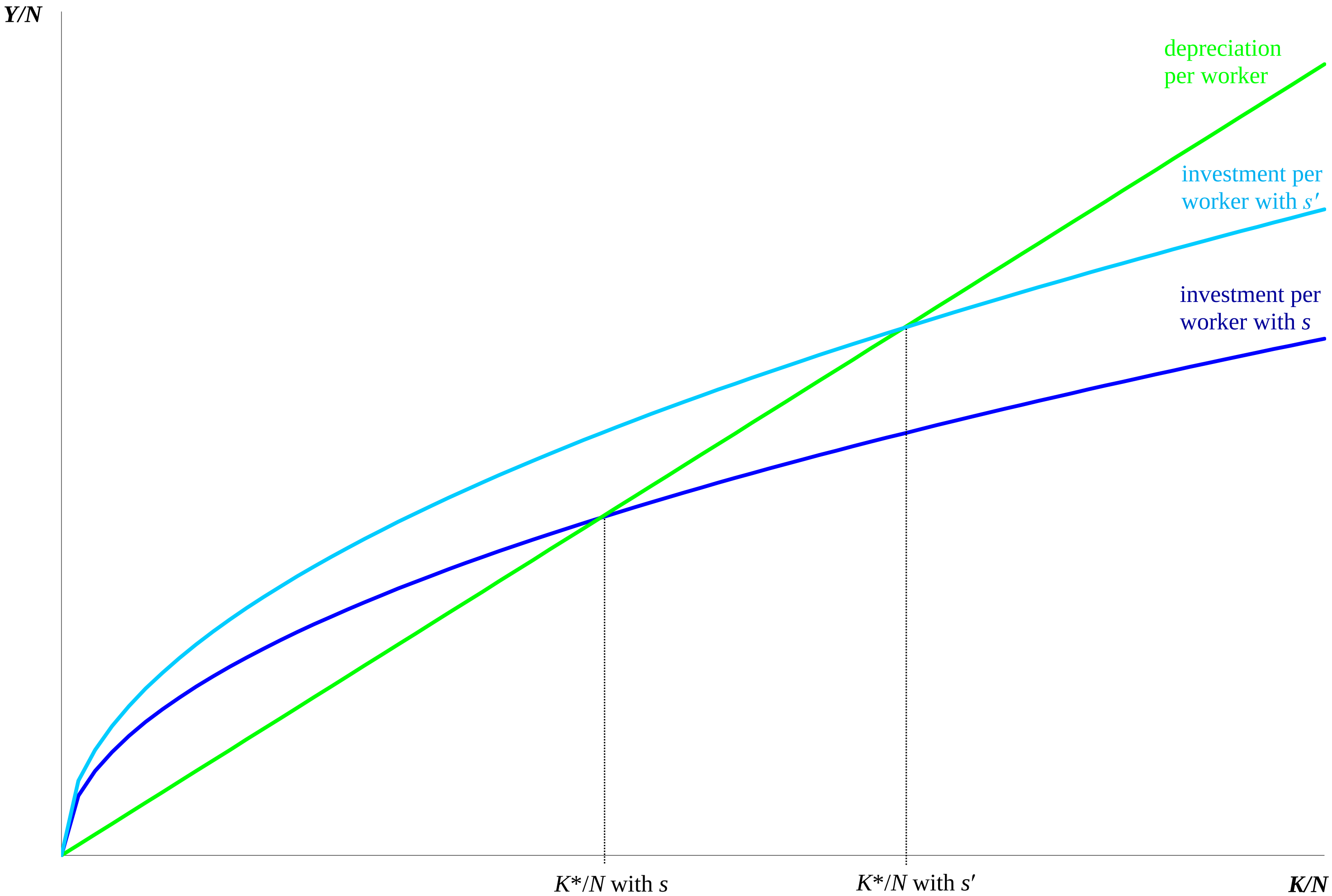
$$\frac{K^*}{N} - \frac{K^*}{N} = sA\sqrt{\frac{K^*}{N}} - \delta\frac{K^*}{N}$$

$$sA\sqrt{\frac{K^*}{N}} = \delta\frac{K^*}{N}$$

$$\frac{sA}{\delta} = \sqrt{\frac{K^*}{N}} \quad \rightarrow \quad \frac{K^*}{N} = \left(\frac{sA}{\delta}\right)^2.$$

b) [8 points] Suppose that the saving rate increases to $s' > s$ but that technology, the depreciation rate and employment stay constant. Show the initial steady state capital-labor ratio and the new steady state capital-labor ratio in a carefully-labeled graph. You do not need to include the production function.

answer:



For full credit on these kinds of graphs in general, the shapes of the curves must be correct, and all axes and curves must be labeled correctly and clearly. It's fine (and probably clearest) to label the functions with words, as is done here, rather than with the symbolic representations of the functions themselves. (For example, the depreciation-per-worker function is just labeled "depreciation per worker" rather than $\delta K/N$.) Unless I ask for the symbolic representations directly, it's best to stick with words when dealing with graphs (though you should obviously be familiar with the symbolic representations by now as well). You definitely don't need to include the expression for the steady state capital-labor ratio (that is, the $(sA/\delta)^2$ that was found in the previous part) in graphs – just label it with the generic K^*/N or in words (the graph above sort of uses a combination), being careful to distinguish which corresponds with which saving rate.

c) [8 points] Assume that the depreciation rate is 5%, the saving rate is 35%, and the level of technology is 25. Calculate the steady state levels of output per worker, investment per worker, depreciation per worker, and consumption per worker.

answer:

$$\frac{Y}{N} = A\sqrt{\frac{K}{N}}, \text{ so } \frac{Y^*}{N} = A\sqrt{\frac{K^*}{N}} = A\left(\frac{sA}{\delta}\right) = A^2\left(\frac{s}{\delta}\right) \rightarrow \frac{Y^*}{N} = (25)^2(0.35/0.05) = 4375.$$

$$I_t = sY_t \rightarrow \frac{I^*}{N} = s\frac{Y^*}{N} = 0.35(4375) = 1531.25.$$

depreciation per worker = $\delta\,\frac{K_t}{N} \rightarrow \delta\,\frac{K^*}{N} = 0.05(0.35(25)/0.05)^2 = 1531.25$ (or you could just have noted that depreciation per worker must be exactly as much as investment per worker in the steady state).

$C = Y - I$ [from equilibrium in the goods market]

$C = Y - sY$

$$\frac{C^*}{N} = (1 - s)\frac{Y^*}{N} = (1 - 0.35)(4375) = 2843.75.$$

[consumption per worker + investment per worker = 2843.75 + 1531.25 = 4375 = output per worker, as it should]

d) [8 points] Continue to assume that the depreciation rate is 5%, the saving rate is 35%, and the level of technology is 25. The economy has been in its corresponding steady state for several periods. Then, in some period *t*, the level of technology suddenly jumps to 30, and stays at this new level permanently. Calculate the level of output per worker in periods *t*-1, *t*, and *t*+1.

answer:

In period *t*-1, the economy is still in its steady state, because nothing has changed yet. So the capital-labor ratio and output per worker are still at their old steady state values. That is, $\frac{Y_{t-1}}{N} = 4375$ (which was found to be the steady state level of output per worker in the previous part).

In period *t*, the capital-labor ratio is going to stay constant at its steady state value (because in period *t*-1, the economy was in its old steady state, so investment was just enough to offset depreciation and thus keep the capital stock constant). However, the change in technology instantaneously increases output per worker even while the capital-labor ratio stays constant. Therefore,

$\frac{K_t}{N} = \frac{K^*}{N} = \left(\frac{sA}{\delta}\right)^2 = (0.35*25/0.05)^2 = 30625$ (using the old level of technology, because we want to find the old steady state value of the capital-labor ratio, and this is determined by the levels of investment and depreciation in the *previous* period); and

$\frac{Y_t}{N} = A\sqrt{\frac{K_t}{N}} = 30*\sqrt{30625} = 5250$ (using the new level of technology in the production function, since the new technology level affects output instantaneously).

The higher output in period *t* will cause saving and therefore investment to be higher in period *t* than it was in period *t*-1, but depreciation will stay the same (because, again, the capital stock has not changed yet). But these investment and depreciation values in period *t* determine the capital-labor ratio in period *t*+1, which will in turn determine the level of output per worker in period *t*+1.

$\frac{K_{t+1}}{N} = \frac{K_t}{N} - \delta\frac{K_t}{N} + s\frac{Y_t}{N} = 30625 - 1531.25 + 0.35*5250 = 30931.25$ (using the same 35% saving rate that has stayed constant throughout, steady state depreciation per worker from the previous part – corresponding with the level of the capital-labor ratio in period *t*, which we have already argued is still at its old steady state level, and the same 5% depreciation rate that has stayed constant throughout – and the level of output per worker in period *t* that was just calculated above).

$\frac{Y_{t+1}}{N} = A\sqrt{\frac{K_{t+1}}{N}} = 30*\sqrt{30931.25} = 5276.185.$

Therefore, the sequence of output per hour that we were required to find is 4375, 5250, 5276.185.

It would be good practice to revisit problem 2d. and the corresponding Figures 4 and 5 from the fifth set of practice problems in the light of these calculations, to get really comfortable with understanding and explaining everything numerically, graphically, and verbally.

Note that the question does NOT ask for the new steady state values, and that these are not necessary to answer the question that is asked. The question is getting at the convergence to the new steady state, but only by looking at the very beginning of this convergence process. The new steady state itself will not be reached for many more periods after the change in the technology level.

2. [15 points total, 2 parts] Consider the full Solow growth model with exogenous technological progress and employment growth. The economy is described by the production function $Y = \sqrt{K}\sqrt{AN}$. Technology, A, grows at 6% per period, and employment, N, grows at 3% per period. The capital stock, K, depreciates at 5% per period, and the economy saves 14% of output, Y, per period. Assume throughout that taxes and government expenditure are zero.

a) [8 points] Calculate the steady state level of capital per effective worker.

answer:

[Note: I made this worth a bit less than 1a) because the general mechanics should be familiar after doing that part, and because I expect a less thorough derivation of the steady state condition in the model with technological progress and employment growth. But you should still show your work.]

$$\frac{Y}{AN} = \frac{\sqrt{K}\sqrt{AN}}{AN} = \sqrt{\frac{K}{AN}}.$$

In the steady state, capital per effective worker will be constant. If investment were zero, capital per effective worker would deteriorate over time for three reasons: depreciation of capital; technology growth; and employment growth. Therefore, if investment adds $0.05K + 0.06K + 0.03K$ to the capital stock, capital will increase by enough to offset all three forces. Therefore, in the steady state,

$$I_t = (0.05 + 0.06 + 0.03)K_t$$

$$\frac{I_t}{A_tN_t} = (0.05 + 0.06 + 0.03)\frac{K_t}{A_tN_t}.$$

$$\frac{0.14Y_t}{A_tN_t} = (0.05 + 0.06 + 0.03)\frac{K_t}{A_tN_t}$$ [because saving equals investment in equilibrium, and saving is 14% of output]

$$0.14\sqrt{\frac{K_t}{A_tN_t}} = (0.05 + 0.06 + 0.03)\frac{K_t}{A_tN_t}$$ [inserting the production function]

$$0.14\sqrt{\left(\frac{K}{AN}\right)^*} = (0.05 + 0.06 + 0.03)\left(\frac{K}{AN}\right)^*$$ [because K/AN is constant in the SS]

$$0.14/(0.05 + 0.06 + 0.03) = \sqrt{\left(\frac{K}{AN}\right)^*} \rightarrow \left(\frac{K}{AN}\right)^* = \left(\frac{0.14}{(0.14)}\right)^2 = 1^2 = 1.$$

b) [7 points] State the rate at which each of the following variables is growing when the economy is in the steady state.

Output ________9%_______

Output per worker ________6%_______

Output per effective worker ________0%_______

Consumption per worker ________6%_______

Investment per effective worker ________0%_______

The saving rate ________0%_______

Depreciation ________9%_______

You should be able to calculate each of these using the growth rate rules, the types of reasoning employed in 1d. of the sixth set of practice problems, and the values given for the growth rates of technology and employment at the beginning of the question. For example,

Depreciation = δK, so the growth rate of depreciation equals the growth rate of δ plus the growth rate of K. The depreciation rate δ is a constant, so its growth rate is zero. In the steady state, the growth rate of K/AN is zero, which implies that the growth rate of K must be equal to the growth rate of AN; and the growth rate of effective workers is equal to the growth rate of technology plus the growth rate of employment. So, the growth rate of Depreciation is equal to the growth rate of technology plus the growth rate of employment, which is 6% + 3% = 9% at the parameter values for this problem.

All that is required for full credit on problems like this is to fill in the blanks with the correct numbers (or symbols if it's a symbolic rather than numerical problem). But for purposes of learning and practice, you should break each down into detailed steps, so that you get used to applying these chains of reasoning, and will be able to go through them more quickly in your head on exams.