EC202C1 – Intermediate Macroeconomic Analysis
Spring 2012, Boston University

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Second Mid-term Exam

Wednesday, April 11, 2012

This is a 50-minute exam. There is a total of 50 points allocated across two questions. Use the number of points allocated to each part as a suggestion for how long to spend on that part. I recommend that you attempt all parts before using more time than is suggested for any one part. If you complete some parts in less than the suggested time, use your extra time to revisit parts you may have had trouble with the first time through and to check your work.

Please read the questions carefully and write your answers in the space provided. You can use the backs of the sheets for scrap paper, but to get full credit you must show all relevant work in the space provided.

Please follow my instructions at all times.

Concentrate and think carefully, but try to relax too!

University ID: Solutions

(Please do not include your name.)

1. [23 points total, 3 parts] Consider the full Solow growth model with exogenous technological progress and employment growth. The economy is described by the production function $Y = \sqrt{K}\sqrt{AN}$. Technology, A, grows at g_A = 5% per period, and employment, N, grows at g_N = 6% per period. The capital stock, K, depreciates at δ = 4% per period, and the economy saves s = 30% of output, Y, per period. Assume throughout that taxes and government expenditure are zero. You can assume that "necessary investment" is given by $(\delta + g_A + g_N)K$ without any derivation or explanation.

a) [10 points] Calculate the steady state level of capital per effective worker.

answer:

First, find the intensive form of the production function:

$$\frac{Y}{AN} = \frac{\sqrt{K}\sqrt{AN}}{AN} = \sqrt{\frac{K}{AN}}.$$

In the steady state, capital per effective worker will be constant, which requires that actual investment must equal "necessary investment":

$$I_t = (0.04 + 0.05 + 0.06)K_t$$

$$\frac{I_t}{A_tN_t} = (0.04 + 0.05 + 0.06)\frac{K_t}{A_tN_t}$$ [divided by effective workers]

$$\frac{0.3Y_t}{A_tN_t} = (0.04 + 0.06 + 0.03)\frac{K_t}{A_tN_t}$$ [because saving equals investment in equilibrium, and saving is 30% of output]

$$0.3\sqrt{\frac{K_t}{A_tN_t}} = (0.04 + 0.05 + 0.06)\frac{K_t}{A_tN_t}$$ [inserting the production function]

$$0.3\sqrt{\left(\frac{K}{AN}\right)^*} = (0.04 + 0.05 + 0.06)\left(\frac{K}{AN}\right)^*$$ [because K/AN is constant in the SS]

$$0.3/(0.04 + 0.05 + 0.06) = \sqrt{\left(\frac{K}{AN}\right)^*} \rightarrow \left(\frac{K}{AN}\right)^* = \left(\frac{0.3}{(0.15)}\right)^2 = 2^2 = 4.$$

b) [8 points] State the rate at which each of the following variables would be growing if the economy were in its steady state. (Your answers should be in numbers, not symbols.)

answer:

The capital stock	______ 11% _____
Capital per effective worker	_______0% ______
Consumption per worker	_______5% ______
The ratio of investment to the capital stock	_______0% ______
Effective workers	______11% ______
The growth rate of employment, g_N	_______0% ______
Saving	______11% ______
The standard of living	_______5% ______

The "standard of living" is defined as output per person, and since there is a background assumption about the proportion of employed people in the population being fixed over time, the analogue in the model is output per worker. I thought this was vocabulary that was used at least a bit in the book and class, but now that I recall, it might have only been in stuff that I said is not required. I'm considering disregarding this question in the grading.

The growth rates of technology and employment are themselves fixed, exogenous parameters; in other words, the growth rate of these growth rates is zero. So, in the steady state and at any other time, the rate at which the growth rate of employment would be growing is zero. Likewise, Saving is equal to the saving rate (a fixed, exogenous parameter) times output; so Saving is always growing at the rate of growth of a constant (i.e. zero) plus the rate of growth of output; and in the steady state, output – and so Saving – would be growing at 11% in this example.

From the condition that actual investment per effective worker equals required investment per effective worker in the steady state, divide through by K/AN, which gives I/K = 0.15. That is, in the steady state, the ratio of investment to the capital stock is a constant, and therefore would be growing at the rate of zero.

c) [5 points] Suppose that this economy has been in its steady state for several periods. Then, the country suffers a war, and half of its capital stock is instantaneously destroyed at the beginning of period *t*. All other aspects of the economy stay the same, and the war ends immediately. Calculate the percentage growth rate of output per worker between periods *t* and *t*+1. (Hint: Technology and employment stay on their predetermined paths, so at the beginning of period *t*, capital per effective worker is exactly half the steady state value you found in the first part. The first thing you need to do is find the level of capital per effective worker in period *t*+1. Start with the standard capital accumulation equation and divide through by $A_{t+1}N_{t+1}$.)

answer:

$$K_{t+1} = K_t - \delta K_t + I_t$$

$$\frac{K_{t+1}}{A_{t+1}N_{t+1}} = (1-\delta)\frac{K_t}{A_{t+1}N_{t+1}} + s\frac{Y_t}{A_{t+1}N_{t+1}}$$

We know what K/AN is in period *t*, but in the denominators on the right hand side, the number of effective workers is for period *t*+1. We have to use the growth rates of technology and employment to convert the denominators to period-*t* terms.

$$\frac{K_{t+1}}{A_{t+1}N_{t+1}} = (1-0.04)\frac{K_t}{A_t(1.05)N_t(1.06)} + (0.3)\frac{Y_t}{A_t(1.05)N_t(1.06)}$$

$$\frac{K_{t+1}}{A_{t+1}N_{t+1}} = \frac{(0.96)}{(1.05)(1.06)}\frac{K_t}{A_tN_t} + \frac{(0.3)}{(1.05)(1.06)}\sqrt{\frac{K_t}{A_tN_t}}$$

Now note that, as of the beginning of period *t*, capital per effective worker is half of its steady state value, i.e. 4/2 = 2. So,

$$\frac{K_{t+1}}{A_{t+1}N_{t+1}} = \frac{(0.96)}{(1.05)(1.06)}2 + \frac{(0.3)}{(1.05)(1.06)}\sqrt{2}$$

= 2.1063.

We can now use this level of capital per effective worker in period *t*+1 to find the level of output per effective worker in period *t*+1:

$$\frac{Y_{t+1}}{A_{t+1}N_{t+1}} = \sqrt{\frac{K_{t+1}}{A_{t+1}N_{t+1}}} = \sqrt{2.1063} = 1.4513.$$

Likewise, $\frac{Y_t}{A_tN_t} = \sqrt{\frac{K_t}{A_tN_t}} = \sqrt{2} = 1.4142.$

So, the percentage growth rate of output per effective worker between periods *t* and *t*+1 is (1.4513 – 1.4142)/1.4142 = 0.0262 = 2.62%. We can now use this along with our growth rate rules to calculate the corresponding growth rate of output per worker, which is what the question asked for:

$g_{Y/AN} = g_Y - g_A - g_N$

in general, and $g_{Y/AN}$ = 2.62% in period *t*+1 in this case, as we just showed. Putting these results together,

$g_Y - g_A - g_N = 0.0262$,

so

$g_Y - g_N = 0.0262 + g_A$;

and

$g_{Y/N} = g_Y - g_N$

by the growth rate rules. So,

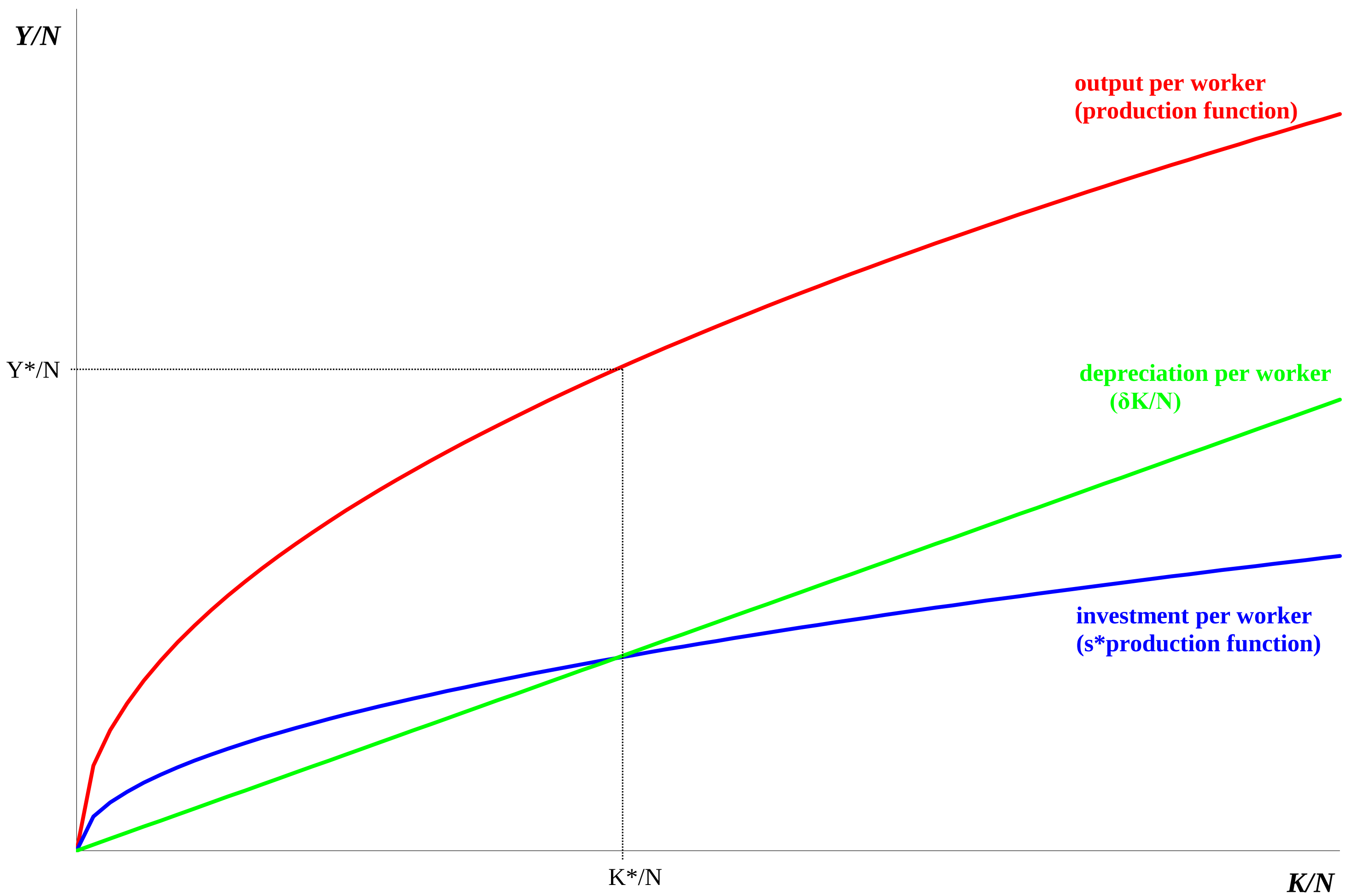
$g_{Y/N} = 0.0262 + g_A = 0.0262 + 0.05 = 0.0762 = 7.62\%$

in period *t*+1. That is, between periods *t* and *t*+1, output per worker will grow at 7.62%. This is greater than 5%, which is the rate at which output per worker would be growing in the steady state. This makes sense, because the destruction of half of the capital stock pushes the economy far below its steady state; and during convergence back to the steady state, capital per effective worker will be growing at a positive (though declining) rate, since investment per effective worker will be exceeding required investment per effective worker. This positive growth of capital per effective worker will lead directly to positive growth of output per effective worker and thus cause the growth rate of output per worker to stay above its steady state rate until convergence is complete and the steady state is re-obtained.

2. [27 points total, 3 parts] Consider the simple Solow model with no technological progress and no employment growth. The economy is described by a production function with the general form $Y = F(A, K, N)$. Technology, A, and the number of workers, N, are constant. The capital stock, K, depreciates at the fixed rate of δ per period, and the economy saves a fixed proportion, s, of output, Y, per period. Assume throughout that taxes and government expenditure are zero. The production function exhibits, as usual, diminishing marginal returns to capital, and constant returns to scale in terms of the capital and labor inputs.

a) [10 points] Draw a graph showing the intensive form of the production function, the depreciation-per-worker function, and the investment-per-worker function. Indicate the position of the steady state capital-labor ratio and the position of the steady state level of output per worker. Be sure to label all axes and curves.

answer:

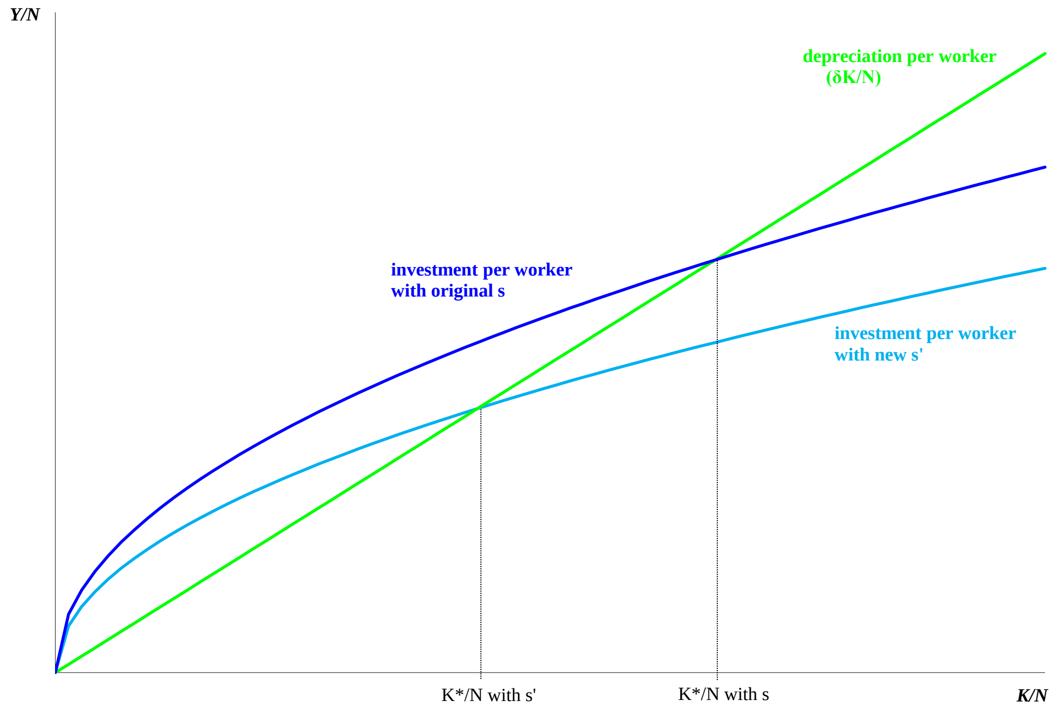


b) [12 points] Suppose that the saving rate decreases to $s' < s$ but that technology, the depreciation rate and employment stay constant.

i. Show the initial steady state capital-labor ratio and the new steady state capital-labor ratio in a carefully-labeled graph. You do not need to include the production function.

ii. Explain briefly how, starting from the initial steady state, the capital-labor ratio will evolve immediately after this change and as the economy converges to its new steady state.

answer:



In the moment that the saving rate changes, the capital-labor ratio is still at its level from the previous period, namely the previous steady state level. Therefore, output per worker and depreciation per worker are also still at their previous steady state levels. In the previous steady state, depreciation per worker was exactly equal to investment per worker, thus keeping the capital-labor ratio constant from period to period (the definition of the steady state). But as soon as the saving rate drops, investment per worker falls below its previous level (because I=S in equilibrium, saving equals the saving rate times output by assumption, and the saving rate is lower while output is the same). The period *after* the change in the saving rate, the capital-labor ratio will hence be lower than its previous steady state level (because depreciation per worker in the previous period outpaced the instantaneously lower investment per worker). This lower level of the capital-labor ratio will be associated with lower investment per worker (because of lower output per worker – the saving rate only changes once, and thereafter stays fixed at the new level s') and lower depreciation per worker. But the latter will still be above the former, so the capital-labor ratio will fall again. This cycle will repeat itself until the capital-labor ratio has converged to its new, lower steady state level, with the rate of convergence slowing down over time as the linear depreciation-per-worker function gets closer to the concave (due to diminishing marginal returns to capital) investment-per-worker function.

c) [5 points] Continue to assume that you do not know the specific form of the production function. However, you are able to observe selected data from this economy, which are presented below. These data indicate, for example, that in some period, the capital-labor ratio was 16,807 and the corresponding level of output per worker was 12,544. The four different observation pairs are from widely-separated periods over a very long time horizon. You know that the production function was stable over this entire time horizon. You also know that the saving rate was always 40%, the depreciation rate was always 20%, and technology and employment were fixed over the entire time horizon. Find the steady state level of consumption per worker in this economy.

K/N	16,807	22,434	32,768	45,531
Y/N	12,544	14,080	16,384	18,688

answer:

The definition of the steady state is a state in which the capital-labor ratio is constant from period to period. This requires that investment per worker is exactly equal to depreciation per worker. We can calculate these two variables for each of the sets of observations we have, because

saving per worker = $s\,\frac{Y_t}{N}$

and

depreciation per worker = $\delta\,\frac{K_t}{N}$,

and we have numbers for all of the component variables. So we can do this, and check to see if investment per worker equals depreciation per worker for any of the four observations we have. If it does, that will identify the steady state levels of the capital-labor ratio and output per worker.

δK/N	3,361.4	4,486.8	6,553.6	9,106.2
sY/N	5,017.6	5,632.0	6,553.6	7,475.2

The four columns of the table above correspond to the same four values of the capital-labor ratio and output per worker in the table given in the question. As indicated, the third column corresponds with the steady state, since investment per worker and depreciation per worker are equal to one another at the associated level of the capital-labor ratio. Thus, the steady state capital-labor ratio in this economy is 32,768.

Now, we can calculate the corresponding level of consumption per worker by first writing consumption in terms of output as we've done before:

$Y = C + I + G$

$Y = C + I$ [because government expenditure is zero by assumption]

$C = Y - I$

$C = Y - sY$ [by our usual assumptions concerning investment]

$\frac{C^*}{N} = (1 - s)\frac{Y^*}{N}$ [simplifying, dividing both sides by N, and evaluating variables at their steady state values]

So, the steady state level of consumption per worker in this economy is

$\frac{C^*}{N} = (1 - 0.4)(16{,}384) = 9{,}830.4.$