Practice Problem Set #2 – Solutions
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Chapter 4

2. a. i=0.05: money demand = $18,000.
i=0.10: money demand = $15,000.

(Just substitute the given nominal income level of $60,000 and the respective interest rate into the given money demand function.)

b. Money demand decreases when the interest rate increases. This makes intuitive sense because bonds (which pay interest) become relatively more attractive to hold as the interest rate increases, so money (which doesn't pay interest) becomes relatively less attractive to hold.

c. If nominal income starts at $60,000 and decreases by 50%, the new level of nominal income will be $30,000. Plugging the interest rate of 10% and nominal income level of $30,000 into the given money demand function gives M^d=$30000(.25)=$7,500. This is half the money demand of $15,000 found in part a. for the same interest rate and the initial nominal income level. Therefore, the demand for money has fallen by 50%.

d. The demand for money falls by 50% in this case too. (Plug $30,000 in for income and 5% in for the interest rate to find that money demand is $9,000. This is half of the $18,000 figure found in part a. for the 5% interest rate and the initial level of nominal income.)

e. A 1% increase in income leads to a 1% increase in money demand. (Likewise, a 1% *decrease* in income will lead to a 1% *decrease* in money demand.) This effect is independent of the interest rate.

4. a.

$M^s=M^d$	(for equilibrium in the market for cash)
$20=$100(.25-i)	(plugging in $100 for income and $20 for M^s, as given)
100i=$25-$20	(rearranging)
i=5/100=5%.	

b. Instead of treating the money supply as a known number, treat it as an exogenous variable called M. Now, impose the condition for equilibrium in the market for cash:

$M^s=M^d$
M=$100(.25-$i$)

This gives an expression that must hold for the market for cash to be in equilibrium. One can insert any interest rate that is desired into this equation, and the resulting value for M will be the required money supply to support the desired interest rate as the equilibrium interest rate.

So if we want the equilibrium interest rate to be 2%, the money supply would have to be M=$100(.25-0.02)=$23; and if we want the equilibrium interest rate to be 12%, the money supply would have to be M=\$100(.25-0.12)=\$13; and, putting these two pieces of information together, if we want the equilibrium interest rate to *increase* by 10 percentage points from 2% to 12%, the money supply would have to *decrease* by \$23-\$13=\$10.

(In general, the equation tells us that any 10-percentage-point increase in i would require a \$10 decrease in the money supply, regardless of the starting interest rate. As another example, note that the required money supply for the equilibrium interest rate to be 15% is \$10; this interest rate is ten percentage points higher than the one found in part a., and the required money supply is \$10 less than the money supply was in that part.)

5. a. Recall that when we think of the demand for cash, we assume that all individuals hold their wealth as either cash or bonds. The demand for bonds is therefore the part of wealth that is left over after the amount of cash held has been subtracted.

$W=M^d+B^d$
$B^d=W-M^d$ (rearranging)
B^d=50,000-60,000(.35-i) (substituting in the given information for W, M^d and $\$Y$)

If the interest rate increases by 10 percentage points, bond demand increases by \$6,000. (You should be able to see this by looking at the equation and reasoning to yourself. To prove it to yourself, choose an arbitrary number for the interest rate and calculate bond demand at that interest rate and again at an interest rate that's ten percentage points higher: if you've done the calculations correctly, you should find that the difference in bond demand is +\$6,000.)

b. An increase in wealth increases bond demand, but has no effect on money demand. (The demand for cash in our model depends only on the interest rate and on income, where the latter is a proxy for the amount of transactions.)

c. An increase in income increases money demand, which hence decreases bond demand, since wealth is constant. (The increase in income does not lead to a change in the level of wealth within the same period.)

d. First of all, the use of "money" in this statement is colloquial. "Income" should be substituted for "money." (There's a very good "Focus Box" near the beginning of Chapter 4 in the textbook called "Semantic Traps" on this point.) Second, when people earn more income, their wealth does not change right away. Thus, they increase their demand for money (in order to be able to complete the additional transactions they want to make), and as a result, in the short run their demand for bonds actually *decreases*.

<u>Chapter 5</u>

2. a. $Y=Z$ (for equilibrium in the goods market, as usual)
$Y=C+I+G$ (substituting in the definition of Z)
$Y=(c_0+c_1Y_D)+I+G$ (substituting in the given consumption function)
$Y=c_0+c_1(Y-T)+I+G$ (substituting in the usual definition of disposable income)
$Y(1-c_1)=c_0-c_1T+I+G$ (isolating all terms containing Y on the left)
$Y^*=[1/(1-c_1)][c_0-c_1T+I+G]$. (solving)

The multiplier is $1/(1-c_1)$.

b. $Y=Z$
$Y=C+I+G$
$Y=(c_0+c_1(Y-T))+(b_0+b_1Y-b_2i)+G$
$Y(1-b_1-c_1)=c_0-c_1T+G+b_0-b_2i$
$Y=[1/(1-b_1-c_1)][c_0-c_1T+G+b_0-b_2i]$

This equation defines the *IS* relation. The multiplier for a given level of the interest rate is $1/(1-b_1-c_1)$. Since the multiplier is larger than the multiplier in part a. (because the denominator is smaller due to the presence of the $-b_1$ term), the effect of a change in autonomous spending is bigger than in part a. An increase in autonomous spending now leads to an indirect increase in investment as well as in consumption.

c. The question gives the *LM* relation, which results from setting the real money supply equal to the real demand for cash, and hence represents equilibrium in financial markets. However, it can be put in a more useful form than the question gives it in. So, the first thing to do is to rearrange the *LM* relation so that i is alone on the left.

$M/P=d_1Y-d_2i$
$d_2i= d_1Y-M/P$
$i=(d_1/d_2)Y-(1/d_2)(M/P)$.

To find equilibrium output, substitute the *LM* expression for the interest rate into the *IS* relation from part b.:

$$Y=\left(\frac{1}{1-b_1-c_1}\right)(c_0-c_1T+G+b_0-b_2[(d_1/d_2)Y-(1/d_2)(M/P)])$$

$$Y\left(1+\frac{b_2(d_1/d_2)}{1-b_1-c_1}\right)=\left(\frac{1}{1-b_1-c_1}\right)(c_0-c_1T+G+b_0+b_2(1/d_2)(M/P))$$

$$Y\left(\frac{1-b_1-c_1+\frac{b_2d_1}{d_2}}{1-b_1-c_1}\right)=\left(\frac{1}{1-b_1-c_1}\right)(c_0-c_1T+G+b_0+b_2(1/d_2)(M/P))$$

$$Y^*=\left(\frac{1}{1-b_1-c_1+\frac{b_2d_1}{d_2}}\right)(c_0-c_1T+G+b_0+(b_2/d_2)(M/P)).$$

The multiplier is $1/(1-b_1-c_1+b_2d_1/d_2)$.

3. a. The *IS* curve shifts left. Equilibrium output and the interest rate fall. The effect on investment is ambiguous because the output and interest rate effects work in opposite directions: the fall in output tends to reduce investment, but the fall in the interest rate tends to increase it.

4. a. $Y=Z$
$Y=C+I+G$

Y=(200+.25(Y-200))+(150+.25Y-1000i)+250
Y-.25Y-.25Y=200-.25(200)+150-1000i+250
0.5Y=550-1000i
Y=1100-2000i (*IS* curve)

b. $(M/P)^s=(M/P)^d$
1600=2Y-8000i
8000i=2Y-1600
i=Y/4000-1/5 (*LM* curve)

c. Substituting from part b. into part a. gives
Y=1100-2000(Y/4000-1/5)
Y+Y/2=1100+400
1.5Y=1500
Y*=1500/1.5=1000.

d. Substituting from part c. into part b. gives
i=(1000)/4000-1/5
i*=0.25-0.2=0.05=5%.

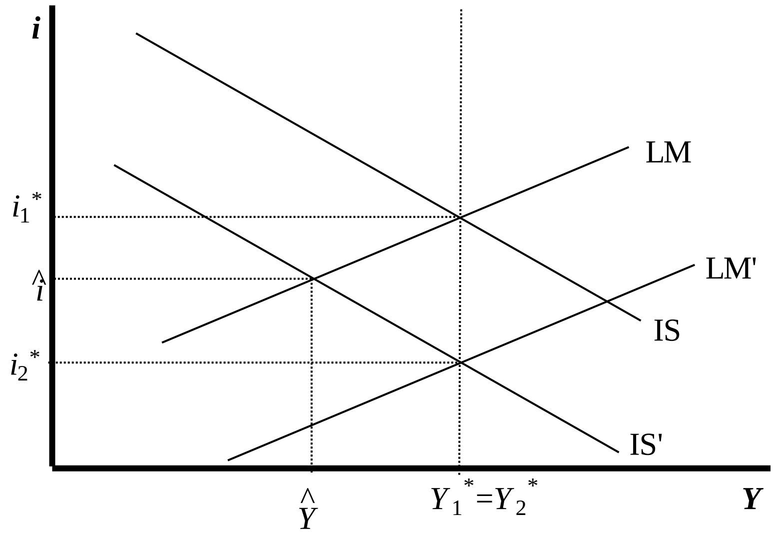
e. Substitute the equilibrium output level and interest rate into the consumption and investment functions. You should get C=400; I=350; G=250 (given); so C+I+G=1000 (i.e. output equals demand, which we would expect, because this is the condition for equilibrium in the goods market that we imposed in deriving the *IS* curve).

f. Derive the *LM* curve over again, with a real money supply of 1,840 rather than 1,600, then re-do parts c., d. and e. using this new *LM* curve and the same *IS* curve as before. You should get i=Y/4000-23/100 as your new *LM* curve; Y*=1040; i*=3%; C=410; I=380. A monetary expansion reduces the equilibrium interest rate and increases equilibrium output. Consumption increases (from 400 to 410 in this example) because output and hence disposable income have increased. Investment increases (from 350 to 380 in this example) because output increases and the interest rate decreases.

g. Derive the *IS* curve over again, with government expenditure of 400 rather than 250, then re-do parts c., d. and e. using this new *IS* curve and the original *LM* curve from part b. You should get Y=1400-2000i as your new *IS* curve; Y*=1200; i*=10%; C=450. A fiscal expansion increases output and the interest rate. Consumption increases (from 400 to 450 in this example) because output and hence disposable income have increased.

8. a. We looked at this case in class, so consult your notes for an accompanying diagram. Increase G (or reduce T, or both), which shifts the *IS* curve to the right; then increase M, which shifts the *LM* curve down.

b. See below for a diagram. Reduce G (or increase T, or both), which shifts the *IS* curve to the left to *IS′*; then increase M, which shifts the *LM* curve down to *LM′*. Overall, the equilibrium switches from the intersection of *IS* and *LM* to the intersection of *IS′* and *LM′* (with the intersection of *IS′* and *LM* representing a theoretical point that is never really reached on the way to the new equilibrium, since we assume that the monetary intervention follows the fiscal intervention almost instantaneously). In the new equilibrium compared to the old one, output is the same (as required) and the interest rate has fallen (by a lot). Investment must have increased, since the interest rate falls while output remains constant.



9. a. The *IS* curve shifts left. Equilibrium output and the interest rate fall.

b. Consumption falls (because output and hence disposable income have fallen). The change in investment is ambiguous: the fall in output tends to reduce investment, but the fall in the interest rate tends to increase investment. The change in private saving equals the change in investment (because investment equals total saving when the goods market is in equilibrium, and public saving has not changed because both *G* and *T* are being held constant). So, private saving could rise or fall in response to a fall in consumer confidence. (When considering the goods market in isolation, we saw in the first set of practice problems that attempts by consumers to increase private saving by lowering autonomous consumption lead, paradoxically, to lower private saving. When we consider the goods market and financial markets simultaneously, as here, this paradoxical result is somewhat weakened: attempts to increase private saving *may* lead to private saving actually increasing.)

<u>Extended Discussion on the Multiplier</u>

Let's take a closer look at the multiplier we found in Chapter 5, problem 2, part c. There, we found that the multiplier is $1/(1-b_1-c_1+b_2d_1/d_2)$. When there is a one-unit change in autonomous expenditure, the multiplier tells us that equilibrium output will increase by this many units. Intuitively, remember that equilibrium output will change for three reasons: it will be pushed up directly by one unit by the initial change in autonomous expenditure (represented by the 1 in the numerator of the multiplier); it will be further pushed up indirectly because consumption rises due to the associated higher disposable income (represented by the $-c_1$ in the denominator of the multiplier); and it will either be partially pulled back or further pushed up by the net change in investment , itself composed of the upward effect due to higher output (represented by the $-b_1$ in the denominator of the multiplier) and the downward effect due to the higher interest rate (represented by the b_2d_1/d_2 in the denominator of the multiplier).

Question 2d. asks under what conditions this multiplier is greater than the one from 2a. (when investment was treated as a purely exogenous variable), which was found to be $1/(1-c_1)$. Mechanically (that is, just setting the first one to be greater than the second and rearranging terms), the multiplier from 2c. is greater than the multiplier from 2a. if the extra terms make the denominator smaller, i.e. if $(-b_1+b_2d_1/d_2)$ is less than zero (or more precisely, $b_1>b_2d_1/d_2$). Intuitively, if the multiplier when both consumption and investment are endogenous is to be larger than the multiplier when only consumption is endogenous, the net change in investment has to be positive, i.e. the positive effect of higher output on investment (again, represented by $-b_1$) must be larger in absolute magnitude than the negative effect of the higher interest rate on investment (again, represented by b_2d_1/d_2).

Note that the multiplier can be less than one, i.e. that a one-unit increase in autonomous expenditure can end up increasing equilibrium output by less than one unit. For this to be the case, the "crowding out" of investment (i.e. the negative effect on investment from the higher interest rate) must be so severe as to more than offset the positive effects from higher output on *both* consumption and investment. Mechanically, this requires that $b_2d_1/d_2>b_1+c_1$. There are, in fact, three scenarios that we could distinguish: 1) crowding out is slight and the net change in investment is positive, so that the multiplier from 2c. is greater than that from 2a.; 2) crowding out is moderate and the net change in investment is negative, so that the multiplier from 2c. is less than that from 2a. but still greater than one; and 3) crowding out is extreme and the net change in investment is large and negative, so that the multiplier from 2c. is not only less than that from 2a. but also less than one. (No matter how extreme crowding out is, though, the multiplier will always be weakly positive. We'll revisit this point briefly in a moment.)

The latter parts of problem 3 from Chapter 5 dwell on the multiplier further. Note carefully, however, that this problem talks about a *drop* in autonomous expenditure (specifically, government spending), whereas we have been talking about an increase until now.

From the *LM* relation, $i=Y(d_1/d_2)-(M/P)/d_2$. To obtain a precise algebraic expression for the equilibrium interest rate, one would substitute Y^* from 2c. into the *LM* relation and proceed with some tedious simplification. But for our purposes, we just need to note that

$I^*= b_0+b_1Y^*-b_2i^*$
$=b_0+(b_1-b_2d_1/d_2)Y^*+(b_2/d_2)(M/P)$.

(The *LM* relation has been substituted in for i and the terms involving Y collected together, with everything evaluated at equilibrium values.)

To obtain a precise algebraic expression for equilibrium investment, one would have to substitute Y^* into this equation and simplify. But we can skip to answering 3e. without any further manipulations. From 2c., holding M/P constant, Y^* decreases by $[1/(1-c_1-b_1+b_2d_1/d_2)]$ when G decreases by one unit. Therefore, from the expression we just found, holding M/P constant, I^* decreases by $(b_1- b_2d_1/d_2)/(1-c_1-b_1+b_2d_1/d_2)$ when G decreases by one unit (which is the amount by which equilibrium investment responds to changes in equilibrium output – $b_1-b_2d_1/d_2$ – multiplied by the amount by which output actually changes following the change in G). So, if G *decreases* by one unit, investment will *increase* if the numerator of this expression is *negative*, i.e. when $b_1<b_2d_1/d_2$. This is the opposite of the condition we arrived at above when discussing 2d., which makes sense, because here we want the interest rate effect on investment (which is positive in this case because the drop in government spending leads to a *lower* interest rate) to *more than offset* the output effect on investment (which is negative in this case because government expenditure and hence output have *fallen*).

To summarize, a fall in G leads to a fall in output (which tends to reduce investment) and to a fall in the interest rate (which tends to increase investment). For investment to increase on net, the interest rate effect (b_2d_1/d_2) must be larger than the output effect (b_1). Note that the interest rate effect is the product of two factors: (i) d_1/d_2, the slope of the *LM* curve, which gives the effect of a one-unit change in equilibrium output on the interest rate; and (ii) b_2, which gives the effect of a one-unit change in the equilibrium interest rate on investment. So it will be larger if either or both of these factors is larger. If money demand falls by a large degree in response to a small drop in income (i.e. d_1 is large, so that the interest rate falls substantially as a result, or in other words, the *LM* curve is steep) and if investment jumps by a large degree in response to any decrease in the interest rate (i.e. b_2 is large, which leads to the overall shift in the *IS* curve being small), then contractionary fiscal policy will lead to a large degree of "crowding in" of investment, so that the net effect on investment will actually be positive.

To continue thinking about this graphically, note that leftward shifts in the *IS* curve (caused, for example, by contractionary fiscal policy) get less and less harmful in terms of lowering equilibrium output as the *LM* curve gets steeper and steeper. On the other hand, when crowding in is large in response to a fiscal contraction, crowding out will likewise be large in response to a fiscal expansion; so *rightward* shifts in the *IS* curve also get less effective at *increasing* equilibrium output as the *LM* curve gets steeper. The most extreme case is when the *LM* curve is perfectly vertical at some given level of output. In this case, shifts of the *IS* curve serve only to change the equilibrium interest rate, and equilibrium output will stay frozen at the given level that the *LM* curve is anchored on. That is, the multiplier will be zero. Crowding in/out of investment is so extreme as to offset all other effects. This requires the parameter d_1 to be arbitrarily large, or in other words for the slope of the *LM* curve to be infinite (which is equivalent to saying that the curve is a vertical line). (And plugging an arbitrarily large value into the multiplier for this parameter, the denominator of the multiplier goes to infinity so that the multiplier itself indeed becomes zero. This is as small as the multiplier can get. We can mathematically make it negative by plugging unwarranted values in for some of the other parameters, but this would lead to negatively-sloped *LM* curves and/or positively-sloped *IS* curves, and all manner of strange predictions that are inconsistent with observed data and the internal structure of the model.)

To make this a bit more concrete, we can return to Chapter 5, problem 4, part g. This part asks you to contemplate an isolated rightward shift of the *IS* curve due to expansionary fiscal policy. Though the question did not ask us to, we can use the output level and interest rate associated with the new equilibrium to find the level of investment that arises in this new equilibrium. Doing so, we find that, remarkably, I=350, which is exactly what we found in 4e. for equilibrium investment before the fiscal policy change. How is that possible? As we've said before, investment is affected in two ways: the increase in output tends to increase investment; and the increase in the interest rate tends to reduce investment. In this very specific example, these two effects exactly offset one another, and investment does not change.

When discussing 3e. above, we found that investment would *increase* in response to a *drop* in government expenditure if $b_1<b_2d_1/d_2$. By the same token, investment will *fall* with an *increase* in government expenditure if this same condition holds, while investment will *rise* with an *increase* in government expenditure if the opposite condition holds, namely $b_1>b_2d_1/d_2$. In the specific numerical example in problem 4, b_1=0.25 (from the investment function) and b_2d_1/d_2=1000(2)/8000=0.25 (from the investment function and the real money demand function). Only because we get the same number for *both* of these calculations does the interest rate effect on investment *exactly* offset the income effect, leading to *zero* net change in investment.

By the way, note that the multiplier derived in 2c. evaluates, in this specific example, to 1/(1-0.25-0.25+0.25)=1/0.75=4/3 (from the investment and money demand functions for the parameters already discussed, and from the consumption function for c_1). Since government expenditure has changed by 400-250=+150 in 4g. relative to 4a.-e., the change in equilibrium output should be the multiplier times this change, or 4/3(150)=200. This corresponds exactly with the original solution to 4g. above, since equilibrium output rose from 1000 to 1200 going from part c. to g., for a change of +200. (I do not recommend memorizing the expression for the multiplier and employing this method on exams. It makes you more vulnerable to memory lapses and algebraic errors, and less able to adapt to examples that do not fit tightly into the form of the behavioral equations from Chapter 5, problem 2. Instead, work directly from the individual market equilibrium conditions, as was done in the solutions to Chapter 5, problem 4 above. The calculations just performed here can be a useful way to check your work when practicing, though.) This is a bit more complicated for 4f. because, if you look at the full expression for equilibrium output in 2c., the real money supply is additionally multiplied by (b_2/d_2), giving us more to keep track of. A similar complication arises if we wanted to consider the effects of a change in taxes. And, of course, everything we've been discussing would become a lot more complicated if we were to let multiple exogenous variables change in different directions and by different magnitudes at the same time.

Finally, let's revisit Chapter 5, problem 9, part b. There has been a leftward shift in the *IS* curve, but this is due to a drop in consumer confidence rather than in government spending. However, the net effect on investment due to a change in autonomous expenditure by one unit is the same whether that change is due to consumer confidence or government spending, and as discussed previously for Chapter 5, problem 3, it is given by the expression $(b_1 - b_2d_1/d_2)/(1-c_1-b_1+b_2d_1/d_2)$. As argued in the original solution to 9b., this change in investment is ambiguous in the general case, and since the change in private saving must be equal to the change in investment for the reasons stated there, the change in private saving is also ambiguous. This extended discussion has explored this ambiguity in much detail. (For example, as discussed above for problem 3, investment will *increase* due to the contraction if $b_1<b_2d_1/d_2$, i.e. if the boost due to the lower interest rate exceeds the drop due to the lower output level. It will decrease if the opposite is true.) Our task here is just to find an expression for the change in private saving and verify that it is indeed the same as the expression for the change in investment.

Private saving is the difference between disposable income and consumption. So the change in private saving is equal to the change in disposable income (which is equal to the change in output since taxes are assumed to remain constant throughout) minus the change in consumption. Let's assume that the drop in consumer confidence was by one unit to keep things simple. Output has thus fallen by this one unit times the multiplier, i.e. by $1/(1-c_1-b_1+b_2d_1/d_2)$. Consumption has fallen for two reasons: it has fallen directly by the one unit that consumer confidence (or autonomous consumption or c_0) has fallen by; and it has fallen indirectly by the marginal propensity to consume times the amount by which disposable income has fallen, i.e. by $c_1/(1-c_1-b_1+b_2d_1/d_2)$. Thus, the overall drop in private saving is given by $1/(1-c_1-b_1+b_2d_1/d_2)-[1+c_1/(1-c_1-b_1+b_2d_1/d_2)]$. Simplifying, this comes out to $(b_1 - b_2d_1/d_2)/(1-c_1-b_1+b_2d_1/d_2)$, which is the same as the expression for the change in investment found previously. (When the contraction is due to a drop in autonomous consumption, as here, the ambiguity in the response of total saving arises because the total drop in consumption could be more or less than the drop in output/disposable income. When, instead, the contraction is due to a drop in government spending or an increase in taxes, *private* saving definitely falls, because consumption only falls indirectly and this indirect fall in consumption is always less than the drop in disposable income. But the change in *total* saving is still ambiguous, because *public* saving has *increased*, and the decline in private saving could be larger or smaller than this increase in public saving.)