

Practice Problem Set #3 – Solutions

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Chapter 6

3. a. $P=(1+\mu)W$ (standard price-setting relation)
 $W/P=1/(1+\mu)$ (rearranging)
 $W/P=1/1.05=0.9524$. (plug in given value for μ)
- b. $W=P(1-u)$ (wage-setting relation, specific form given)
 $u=1-W/P$ (divide both sides by P and solve for u)
 $u_n=1-0.9524=0.0476=4.76\%$. (plug in value for W/P found above)

What we've done in these two parts is found equilibrium in the labor market, i.e. the unemployment rate that arises when the real wage demanded by workers is exactly equal to the real wage paid by firms. The actual price level appears in the specific form of the wage-setting relation given by the problem instead of the expected price level, so the implicit assumption is that workers perfectly anticipate the price level in this specific case. When there is no confusion over the price level (i.e. the expected price level equals the actual price level), the economy is in its natural state. The equilibrium unemployment rate when the economy is in its natural state is called, simply, the natural rate of unemployment.

- c. $W/P=1/1.1=0.91$; $u=1-0.91=0.09=9\%$. The increase in the markup lowers the real wage. Algebraically, from the wage-setting equation, the unemployment rate must rise for the real wage to fall. So the natural unemployment rate increases. Intuitively, an increase in the markup implies more market power for firms, and therefore a higher price level for a given nominal wage. This will lead to less production, as the deterioration in real wages will push some workers to choose unemployment. (The overall effect is consistent with something you might have learned in a micro class: monopolists and oligopolists use their market power to increase the price of goods by reducing their supply, i.e. prices are higher and production lower than with perfect competition.) Less production implies less demand for labor all other things equal, so the natural unemployment rate rises.

Extra Question

Consider the following behavioral equations and information describing the economy:

goods market

$$C = 200 + 0.25Y_D$$

$$I = 150 + 0.25Y - 1000i$$

$$G = 250$$

$$T = 200$$

financial market

$$(M/P)^d = 2Y - 8000i$$

$$M^s = 1600$$

$$[(M/P)^s = 1600/P]$$

labor market

$$W = P^e(0.815 - 0.315u) \quad [\text{wage-setting relation}]$$

$$P = 1.25W \quad [\text{price-setting relation - i.e. 25\% mark-up on wages}]$$

$$u = (1 - Y/1050) \quad [\text{relationship linking the unemployment rate and output}]$$

a) Calculate the Aggregate Demand curve. (Derive the *IS* curve as in Chapter 5, problem 4. Derive the *LM* curve as in Chapter 5, problem 4, but treating the aggregate price level *P* as an arbitrary variable. Eliminate the interest rate as usual when equating *IS* and *LM*. Arrange the resulting expression so that *P* is alone on the left and a ratio including *Y* and some numbers is on the right.)

answer:

$$Y = Z$$

$$Y = C + I + G$$

$$Y = (200 + 0.25(Y - 200)) + (150 + 0.25Y - 1000i) + 250$$

$$Y - 0.25Y - 0.25Y = 200 - 0.25(200) + 150 - 1000i + 250$$

$$0.5Y = 550 - 1000i$$

$$Y = 1100 - 2000i \quad (IS \text{ curve})$$

$$(M/P)^s = (M/P)^d$$

$$1600/P = 2Y - 8000i$$

$$8000i = 2Y - 1600/P$$

$$i = Y/4000 - 1/5P \quad (LM \text{ curve})$$

Substituting *LM* into *IS* gives

$$Y = 1100 - 2000(Y/4000 - 1/5P)$$

$$1.5Y = 1100 + 400/P$$

$$P(1.5Y - 1100) = 400$$

$$P = 400/(1.5Y - 1100) \quad (AD \text{ curve}).$$

b) Calculate the Aggregate Supply curve. (Convert the wage-setting relation so that it depends positively on output rather than negatively on the unemployment rate. Then insert the wage-setting relation into the price-setting relation, treating the expected price level P^e as an arbitrary variable. Use up to six decimal places as necessary.)

answer:

Substituting the linking relationship into the wage-setting relation gives

$$W = P^e(0.815 - 0.315(1 - Y/1050))$$

$$W = P^e(0.815 - 0.315 + 0.315Y/1050)$$

$$W = P^e(0.5 + 0.0003Y)$$

Substituting this into the price-setting relation gives

$$P = 1.25(P^e(0.5 + 0.0003Y))$$

$$P = P^e(0.625 + 0.000375Y) \quad (AS \text{ curve}).$$

c) Find the natural rate of unemployment and the natural level of output. (Remember that the “natural” state of the economy occurs when $P = P^e$. You should find that the natural level of output is 1000. The natural rate of unemployment will not be a round number. You can either find output first through the AS curve then unemployment through the linking relationship, or the unemployment rate first by equating real wages from the wage-setting and price-setting relations then output through the linking relationship.)

answer:

Starting from the Aggregate Supply curve, set $P^e = P$ and divide both sides by P :

$$P = P^e(0.625 + 0.000375Y)$$

$$P = P(0.625 + 0.000375Y)$$

$$1 = 0.625 + 0.000375Y$$

$$0.000375Y = 1 - 0.625$$

$$Y_n = 0.375/0.000375 = 1000.$$

Now substitute the natural level of output into the linking relation to find the natural rate of unemployment:

$$u = (1 - Y/1050)$$

$$u_n = (1 - 1000/1050) = 0.0476 = 4.76\%.$$

The alternative (and equivalent) route is to set $P^e = P$ in the wage-setting relation, solve both the wage-setting and price-setting relations for the real wage W/P , and impose equilibrium in the labor market:

$$(W/P)^{WS} = (W/P)^{PS}$$

$$0.815 - 0.315u = 1/1.25$$

$$0.315u = 0.815 - 0.8$$

$$u_n = 0.015/0.315 = 0.0476 = 4.76\%.$$

(This is equivalent to what we did for Chapter 6, problem 3, parts a. and b., though it’s a bit of a coincidence that we get the same numerical answer, since the wage-setting and price-setting relations are different here.) If you plug this into the linking relationship you should get the same answer of 1000 for the natural level of output, though you have to carry all the decimal places to get it exactly correct.

d) Assume that the expected price level is 1.1. Calculate the short-run equilibrium output level and price level. Is the equilibrium output level higher or lower than the natural level? Is this

what you would expect? Explain. (Equate AS and AD , and solve for Y . This will be tedious, and will not result in a round number. You will end up having to use the quadratic formula. Use six or more decimal places in your intermediate calculations.)

answer:

$$\begin{aligned} AS &= AD \\ P^e(0.625 + 0.000375Y) &= 400/(1.5Y - 1100) \\ 1.1(0.625 + 0.000375Y)(1.5Y - 1100) &= 400 \\ 1.1(0.9375Y - 687.5 + 0.0005625Y^2 - 0.4125Y) &= 400 \\ 0.00061875Y^2 + 0.5775Y - 1156.25 &= 0. \end{aligned}$$

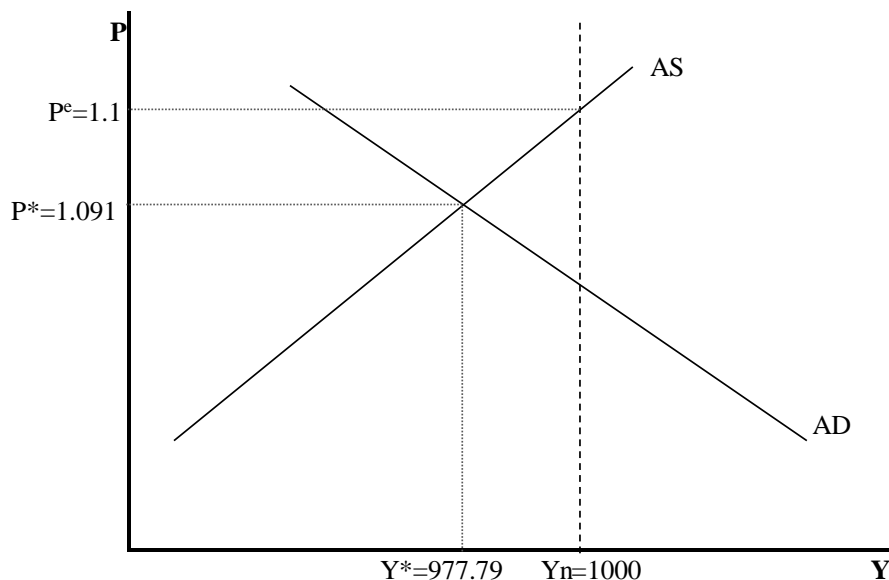
This gives a quadratic in Y , and unfortunately, that's about as pretty as it's going to get. There's nothing else to do except crank it through the quadratic formula:

$$\begin{aligned} Y &= (-0.5775 \pm \sqrt{0.5775^2 - 4*(0.00061875)*(-1156.25)})/(2*(0.00061875)) \\ Y^* &= 977.79 \quad (\text{or } Y = -1911.13, \text{ which can be discarded}). \end{aligned}$$

This can be plugged into either the aggregate demand or aggregate supply curve to get the actual price level:

$$\begin{aligned} P^* &= 400/(1.5Y^* - 1100) \\ P^* &= 400/(1.5(977.79) - 1100) \\ P^* &= 400/366.685 = 1.091. \end{aligned}$$

We are therefore in a situation in which the actual price level (1.091) is less than the expected price level (1.1). In this situation, we'd expect output to be less than the natural level, and this is what we've found ($977.79 < 1000$). When workers expect the price level to be higher than it actually is, they think that their real wage is lower than it actually is. This low expected real wage corresponds with a high unemployment rate, which limits the output that firms can produce.



e) Continue to assume that the expected price level is 1.1. Calculate the unemployment rate in the short-run equilibrium. Calculate the real wage rate that workers expect to earn, and calculate the real wage rate that is actually paid.

answer:

$$u^* = (1 - Y^*/1050)$$

$$u^* = (1 - 977.79/1050) = 0.0688 = 6.88\%.$$

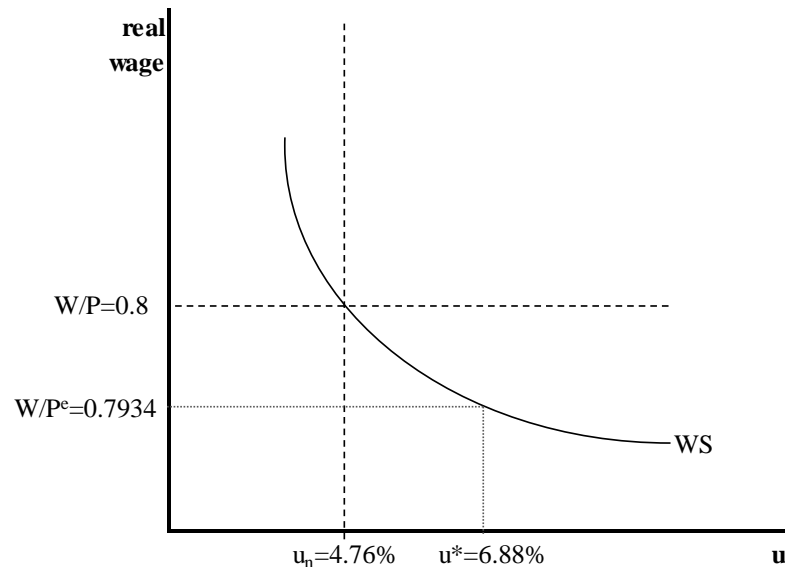
$$W = P^e(0.815 - 0.315u)$$

$$W = 1.1(0.815 - 0.315(0.0688)) = 0.8727.$$

$$W/P^e = 0.8727/1.1 = 0.7934.$$

$$W/P = 0.8727/1.091 = 0.8.$$

(The actual real wage is always $1/1.25 = 0.8$, from the price-setting relation. The last calculation above will only come out exactly right if you've carried all of the decimal places through all of the calculations.)



f) Repeat parts d) and e), this time assuming that the expected price level is 0.9.

answer:

$$AS = AD$$

$$P^e(0.625 + 0.000375Y) = 400/(1.5Y - 1100)$$

$$0.9(0.625 + 0.000375Y)(1.5Y - 1100) = 400$$

$$0.9(0.9375Y - 687.5 + 0.0005625Y^2 - 0.4125Y) = 400$$

$$0.00050625Y^2 + 0.4725Y - 1018.75 = 0.$$

$$Y = (-0.4725 \pm \sqrt{0.4725^2 - 4*(0.00050625)*(-1018.75)})/(2*(0.00050625))$$

$$Y = 1026.69 \quad (\text{or } Y = -1960.03, \text{ which can be discarded}).$$

$$P = 400/(1.5Y - 1100)$$

$$P = 400/(1.5(1026.69) - 1100)$$

$$P = 400/440.035 = 0.909.$$

We are therefore in a situation in which the actual price level (0.909) is greater than the expected price level (0.9). In this situation, we'd expect output to be greater than the natural level, and this is what we've found ($1026.69 > 1000$). When workers expect the price level to be lower than it actually is, they think that their real wage is higher than it actually is. This high expected real wage corresponds with a low unemployment rate, which allows firms to produce at a high capacity. (Graphically, aggregate supply intersects aggregate demand to the *right* of the natural level of output in this case.)

$$u = (1 - Y/1050)$$

$$u = (1 - 1026.69/1050) = 0.0222 = 2.22\%.$$

$$W = P^e(0.815 - 0.315u)$$

$$W = 0.9(0.815 - 0.315(0.0222)) = 0.7272.$$

$$W/P^e = 0.7272/0.9 = 0.808.$$

$$W/P = 0.7272/0.909 = 0.8.$$

g) Explain how, over the medium run, the economy will converge to its natural state, starting either from part d) or part f). Calculate the price level and the real wage rate in the natural state.

answer:

The natural level of output is 1000, and the associated price level can be found by plugging this into the Aggregate Demand curve:

$$P = 400/(1.5Y - 1100)$$

$$P_x = 400/(1.5(1000) - 1100)$$

$$P_x = 400/400 = 1.$$

The natural state will be reached when price expectations have converged to the actual price level, so we also know that $P_x^e = 1$ when the medium run equilibrium is eventually reached.

The real wage (which will be equal to the expected real wage when price expectations are aligned with the actual price level once the natural state is obtained) is always 0.8 (from the price-setting relation, though this can also be confirmed for the natural state by plugging the expected price level of 1 and the natural rate of unemployment into the wage-setting relation).

This is the point that the economy will converge to over the medium run in both cases.

Expected Price (P^e)	Equilibrium Output (Y)	Actual Price in Equilibrium (P)	Nominal Wage (W)	Expected Real Wage (W/P^e)	Actual Real Wage (W/P)	Unemployment Rate (u)
1.1	977.79	1.0908	0.8727	0.7933	0.8	0.0688
1.05	988.41	1.0454	0.8363	0.7965	0.8	0.0587
1.025	994.08	1.0227	0.8182	0.7982	0.8	0.0533
1.0125	997.00	1.0114	0.8091	0.7991	0.8	0.0505
1.00625	998.49	1.0057	0.8045	0.7995	0.8	0.0491
1.003125	999.24	1.0028	0.8023	0.7998	0.8	0.0483

The table above may help to make the explanation more concrete. The first column is a completely imaginary sequence of adjustments to the expected price level. We don't have an exact model for how price expectations are formed, so this has to be an arbitrary choice. (An alternative might have been to take the actual price from the previous period as the next period's expectation; this works fine, but convergence to the natural state of the economy takes many more periods.) The columns to the right are calculated using the same steps from the parts above.

Starting from part d), where the expected price level is higher than the actual price level and the economy is hence producing below the natural level of output, it is reasonable to suppose that workers will eventually adjust their price expectations downwards. When this happens, the resulting higher expected real wage will draw more workers out of unemployment, and hence output will rise. (The workers who have a job are pleasantly surprised every time they go to the store and are able to buy more goods than they thought they'd be able to, so they might not be so quick to change their expectations and hence accept a lower nominal wage. However, there is a large number of potential workers unemployed at the low expected real wage who will soon realize that, if they get a job, the wage will buy more than they previously thought, and who will then join the workforce.) There are two forces on nominal wage demands (as seen in the wage-setting relation): a lower expected price level pushes nominal wage demands down; while higher output (and hence a lower unemployment rate) improves the bargaining position of existing workers and pushes nominal wage demands up. The first effect dominates, so the nominal wage falls, which leads firms to set a lower price level. (We know that the expected *real* wage must rise, though, because the unemployment rate has fallen as output has risen.) The new price level is thus below the new expected price level once again, which will prompt another downward revision of price expectations and hence another cycle of wage, output and price adjustments. This process will continue until the economy converges to the natural level of output, at which point price expectations will be exactly in line with the actual price level.

Expected Price (P^e)	Equilibrium Output (Y)	Actual Price in Equilibrium (P)	Nominal Wage (W)	Expected Real Wage (W/P^e)	Actual Real Wage (W/P)	Unemployment Rate (u)
0.9	1026.69	0.9090	0.7272	0.8080	0.8	0.0222
0.95	1012.70	0.9545	0.7636	0.8038	0.8	0.0355
0.975	1006.20	0.9773	0.7818	0.8019	0.8	0.0417
0.9875	1003.07	0.9886	0.7909	0.8009	0.8	0.0447
0.99375	1001.52	0.9943	0.7955	0.8005	0.8	0.0462
0.996875	1000.76	0.9972	0.7977	0.8002	0.8	0.0469

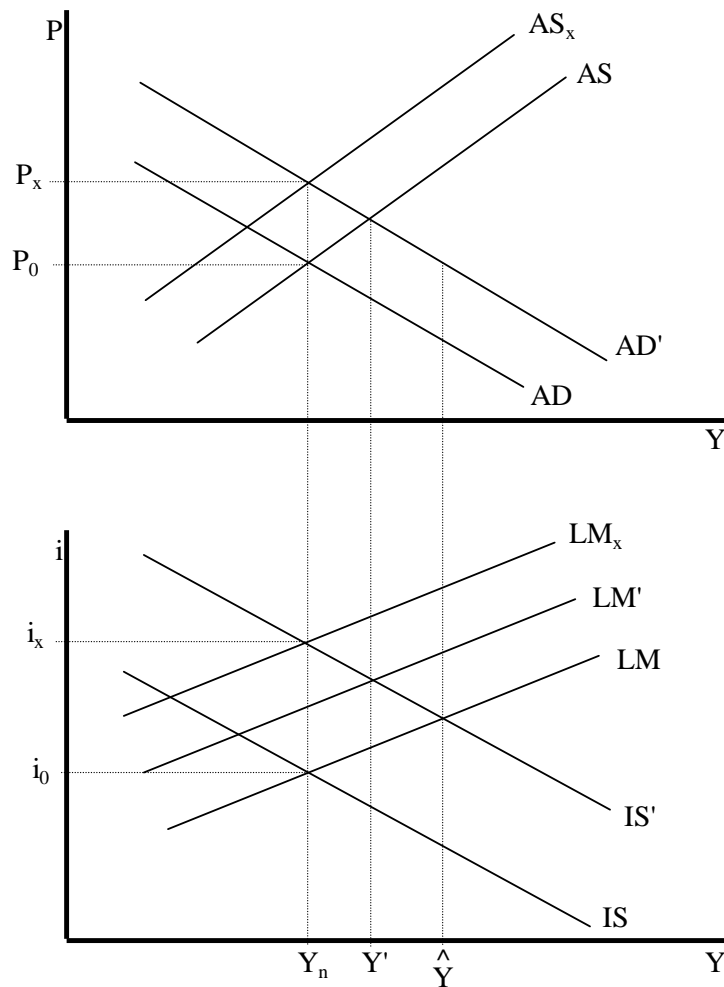
Starting from part f), where the expected price level is lower than the actual price level and the economy is hence producing above the natural level of output, it is reasonable to suppose that workers will eventually adjust their price expectations upwards. When this happens, the resulting lower expected real wage will push more workers into unemployment, and hence output will fall. (We might suppose that expectations will adjust more rapidly in this case, because workers are always getting to the store and realizing they can buy fewer goods with their paychecks than they thought they'd be able to.) There are two forces on nominal wage demands (as seen in the wage-setting relation): a higher expected price level pushes nominal wage demands up; while lower output (and hence a higher unemployment rate) deteriorates the bargaining position of existing workers and pushes nominal wage demands down. The first effect dominates, so the nominal wage rises, which leads firms to set a higher price level. (We know that the expected *real* wage must fall, though, because the unemployment rate has risen as output has fallen.) The price level is thus above the expected price level once again, which will prompt another upward revision of price expectations and hence another cycle of wage, output and price adjustments. This process will continue until the economy converges to the natural level of output, at which point price expectations will be exactly in line with the actual price level.

Chapter 7

2. This is very much like Figure 10 in Chapter 7 of the text, except it deals with a fiscal expansion rather than contraction. Notation in the following responses refers to the graph on the next page.
 - a. IS shifts right to IS' , causing a shift from AD to AD' . Initially, output wants to jump up to \hat{Y} , but as it starts to expand, more workers are hired, leading to higher nominal wage demands and hence a higher aggregate price level. This causes a deterioration in the real value of the money supply, so LM shifts up to LM' . The short run equilibrium is therefore at Y' . But here, price expectations (which initially stay stuck at the old actual price level P_0 from the original equilibrium) are below the new actual price level. So price expectations will adjust upwards, putting further upward pressure on prices and causing an upward shift in AS and LM . This will start a cycle of adjustments, which will end when the aggregate supply curve has shifted all the way up to AS_x , where it intersects AD' at the

natural level of output. Correspondingly, LM will continue to shift up as the price level increases and the real value of the money supply hence deteriorates; it will shift all the way to LM_x , where it intersects IS' at the natural level of output.

- b. Output returns to its unchanged natural level. The interest rate and the price level increase. Consumption is higher than it was before the tax cut, because output is the same but taxes are lower. Investment is lower because output is the same and the interest rate is higher. Because output is the same and government expenditure is unchanged (it's not mentioned, so it's implicitly assumed to be held constant), the magnitude of the increase in consumption must be exactly equal to the magnitude of the drop in investment.



4. a. Money is neutral in the sense that the nominal money supply has no effect on output or the interest rate in the medium run. Output returns to its natural level. The interest rate is determined by the position of the IS curve and the natural level of output. However, despite the neutrality of money in the medium run, an increase in the money supply will increase output and reduce the interest rate in the short run. Therefore, expansionary monetary policy can be used to speed up

the economy's return to the natural level of output when output starts below the natural level.

- b. In the medium run, fiscal policy affects the interest rate and hence investment, so fiscal policy is not considered neutral.
- c. False. Simple changes in general government expenditure and/or the nominal money supply do not change the natural rate of unemployment and hence also do not change the natural level of output. However, policies such as labor market reforms – e.g. changes in the degree of unemployment insurance – can affect the natural rate of unemployment and hence the natural level of output. (This takes us into the domain of productivity-related aggregate supply shifts, which we have not covered yet in class and which will not be included on the first mid-term.)
5. The picture looks exactly like Figure 10 in Chapter 7 of the text. The initial source of the drop to IS' (and hence to AD' as well), instead of contractionary fiscal policy as in the textbook example, is a drop in autonomous consumption, which we have represented with the parameter c_0 in our treatment of the goods market.

- a. *SR*: short run (including initial price adjustment due to increased unemployment);
MR: medium run

	IS	LM	AD	AS
SR	left	down a bit	left	no change
MR	same as SR	down further	same as SR	down

b-c.

	Y	i	P
SR	falls	falls	falls
MR	back to original Y_n	falls further	falls further

	C	I	Private S
SR	falls	ambiguous	ambiguous
MR	Increases from SR but still lower than original level	rises (above original level)	rises (above original level)

In the medium run, consumption must be lower than its original level because disposable income is unchanged (since output returns to its natural level and taxes haven't changed) but consumer confidence is lower. Unchanged disposable income combined with lower

consumption means that private saving has increased (comparing the new medium-run equilibrium to the original starting point).

The short-run change in investment is ambiguous, because the interest rate falls, which tends to increase investment, but output also falls, which tends to reduce investment (which should be a very familiar argument by now). But in the medium run, investment must rise because the interest rate falls but output returns to its original level. (This is compared to the original starting point. Investment also must rise compared to its short-run level, since output increases and the interest rate falls as the economy converges from here to its new medium-run equilibrium.)

Since the budget deficit does not change in this problem at any point, the change in private saving equals the change in investment. It is possible that private saving (and hence investment) will fall in the short run. But private saving (and hence investment) must rise in the medium run by the foregoing arguments. That is, the paradox is resolved: consumers decide to try to save more, and by the time the economy converges to its new medium-run equilibrium, they actually are saving more.

8.
 - a. The AD curve shifts left in the short run. Output and the price level fall in the short run. In the medium run, the expected price level falls, and AS shifts right, returning the economy to the original natural level of output, but at a lower price level. (See Figure 9 in Chapter 7 of the text. The cause of the shift to AD' , instead of contractionary fiscal policy as in the textbook example, is a deterioration in business confidence, which we have represented with the parameter b_0 in our treatment of the goods market.)
 - b. The unemployment rate rises in the short run (because output falls), but returns to its original level (the natural rate, which is unchanged) in the medium run.

(NB: the textbook mis-labels the next three parts; the second batch of parts named a.-c. in the text should actually be named c.-e.)

- c. The Fed should increase the money supply, which shifts the AD curve right. A monetary expansion of the proper size exactly offsets the effect of the decline in business confidence on the AD curve. The net effect is that the AD curve effectively does not move, and neither does the AS curve since price expectations don't change in the short run and will again be aligned with the original actual price level after the expansionary monetary policy.
- d. Under the policy option in part c., output and the price level are higher in the short run than they would have been in part a. without a policy response. In the medium run, output is the same in parts a. and c. (because it's at the natural level – which hasn't changed – in both cases), but the price level is higher in part c.
- e. The unemployment rate is lower in the short run in part c. than when there wasn't a policy response as in part b. In the medium run, the unemployment rate is the same in parts b. and c. (because it's at the natural rate – which hasn't changed – in both cases).