Practice Problem Set #5
EC202C1, Spring 2012, Jeremy Smith

1. Consider a simple version of the Solow growth model, with no technology growth and no growth in employment. Suppose that the economy is described by the production function $Y = A\sqrt{K}\sqrt{N}$, where Y is output, K is the level of the capital stock, N is the (constant) number of workers, and A is the (constant) level of technology. Assume throughout that taxes and government expenditure are zero.

a) Convert the production function to intensive form (i.e. find an expression for output per worker).

b) Write down the standard equation describing how the capital stock evolves over time, using δ for the depreciation rate. From this, derive an expression for the change in the capital-labor ratio between years t and t+1 that depends only on the level of the capital-labor ratio in year t and parameters, using s for the saving rate. Explain your steps.

c) Find an expression for the steady state capital-labor ratio and the steady state level of output per worker. Show your work.

d) Find an expression for the steady state level of investment per worker and the steady state level of depreciation per worker. Do your answers make sense in relation to each other? Explain.

e) Find an expression for the steady state level of consumption per worker.

2. Consider a simple version of the Solow growth model, with no technology growth and no growth in employment. Suppose that the economy is described by the production function $Y = A\sqrt{K}\sqrt{N}$, where Y is output, K is the level of the capital stock, N is the (constant) number of workers, and A is the level of technology (which is constant from period to period but can experience one-time changes exogenously). Assume throughout that taxes and government expenditure are zero.

a) Draw a graph showing the intensive form of the production function for some technology level A_1, the depreciation-per-worker function for some depreciation rate δ_1, and the investment-per-worker function for some saving rate s_1. Label the associated steady state capital-labor ratio and steady state level of output per worker.

b) Suppose that the saving rate increases to $s_2 > s_1$ but that the technology level and depreciation rate stay constant at A_1 and δ_1 respectively. Illustrate the initial steady state and the new steady state in a graph without the production function. Describe briefly how the economy will converge to the new steady state.

c) Forget about the change in the saving rate. Suppose that the depreciation rate rises to $\delta_2 > \delta_1$ but that the technology level and saving rate stay constant at A_1 and s_1 respectively. Illustrate the initial steady state and the new steady state in a graph without the production function. Describe briefly how the economy will converge to the new steady state.

d) Forget about the change in the depreciation rate. Suppose that the technology level rises to $A_2 > A_1$ but that the depreciation rate and saving rate stay constant at δ_1 and s_1 respectively. Illustrate the initial steady state and the new steady state in a graph without the production function. Describe briefly how the economy will converge to the new steady state. (Though similar on the surface to part b), be careful to understand the differences. You might want to draw the graph again including the production function.)

e) State how the level of output per worker in the new steady state compares with that in the initial steady state for each of these three scenarios. Make sure this accords both with your intuition and with the algebraic expression you found for steady state output per worker in the first problem.

3. Consider a simple version of the Solow growth model, with no technology growth and no growth in employment. Suppose that the economy is described by the production function $Y = A\sqrt{K}\sqrt{N}$, where Y is output, K is the level of the capital stock, N is the (constant) number of workers, and A is the (constant) level of technology. Assume throughout that taxes and government expenditure are zero, and that the level of technology, A, is fixed at a value of 4.

a) Calculate the level of output corresponding to a capital stock of 250 units and employment of 250 workers. Calculate the level of output corresponding to a capital stock of 500 units and employment of 500 workers. Does this production function exhibit constant returns to scale? Explain.

b) Hold employment constant at 250 workers. Calculate the difference between the levels of output corresponding to capital stock levels of 250 and 300 units. Calculate the difference between the levels of output corresponding to capital stock levels of 500 and 550 units. Does this production function exhibit diminishing returns to capital? Explain.

c) Assume that the depreciation rate is 10% (i.e. $\delta = 0.1$). Calculate the steady state capital-labor ratio and the steady state levels of output per worker and consumption per worker for saving rates of 25%, 50% and 75%. (You can use the expressions you derived in the first problem, but it would be good practice to derive them again from the capital accumulation equation and the specific production function of this problem. Using memorized expressions on tests and exams earns very little partial credit and isn't a good way to learn. To think visually about what is going on, think back to part b) of problem 2, perhaps adding the production function to the graph.)

d) Continue to assume that the depreciation rate is 10%. Suppose that the economy has been in the steady state corresponding to a saving rate of 50% for a number of periods. Then, in some period t, the saving rate suddenly and permanently falls to 25%. Calculate the capital-labor ratio in periods t+1, t+2 and t+3.