

EC371 – Environmental Economics, Fall 2010, Boston University

Instructor: Jeremy Smith

Final Exam

Monday, December 20, 2010

This is a 108-minute exam, but you will have 120 minutes to complete it. There is a total of 108 points allocated across four questions. In addition, there is one bonus question at the end. Use the number of points allocated to each part as a rough guide to how long to spend on that part. I recommend that you use one minute per point *at most* until you have gotten through each question, then use your extra time to revisit parts you may have missed the first time through and to check your work.

Please read the questions carefully and write your answers in the space provided. You can use the backs of the sheets for scrap paper, but to get full credit you must show all relevant work in the space provided.

Please follow my instructions at all times.

Concentrate and think carefully, but try to relax too!

Student Number: Solutions

(Please do not include your name.)

1. [3 parts, 36 points total] Suppose that a tannery is situated on the outskirts of a small town. The private marginal cost (MC) of producing tanned hides (in thousands of dollars per shipment) is given by $MC = 3 + 2.25Q$ where Q is shipments. In addition to the private marginal cost, an external cost is incurred. Tanning causes the discharge of toxic chemicals which, through contamination of the town's water supply, cause damage valued at $MD = 1.8Q$ (also in thousands of dollars per shipment). Even though we are concerned with a single firm, it behaves perfectly competitively. The aggregate inverse demand curve for tanned hides (in thousands of dollars per shipment), representing both the private and social marginal benefits (MB), is given by $MB = 27 - 0.75Q$.

a) [16 points] Find both the market equilibrium and socially efficient quantities of shipments of tanned hides. Calculate the dollar value of the efficiency gain associated with production of the efficient rather than the market quantity.

answer:

market: $MB = MC$
 $27 - 0.75Q = 3 + 2.25Q$
 $3Q = 24$
 $Q^* = 24/3 = 8.$

efficient: $MB = MSC$
 $MB = (MC + MD)$
 $27 - 0.75Q = [(3 + 2.25Q) + (1.8Q)]$
 $4.8Q = 24$
 $Q^{**} = 24/4.8 = 5.$

The efficiency gain from moving from the market to the efficient quantity simply comes from removing (i.e. avoiding) the efficiency loss associated with the market outcome. The relevant area is that of the triangle formed between the MSC and MB curves, from their intersection at the efficient quantity of five up to the market quantity of 8. If you check this – perhaps with the aid of a graph – you should get

$$\text{gain} = 1/2 * (8 - 5) * (35.4 - 21) = \$21.6 \text{ (thousand).}$$

[6 of the 16 points were for calculating the efficiency gain. I was fairly picky with awarding these, as it's an important concept that was stressed repeatedly in class.]

b) [12 points] Suppose that the town council has crafted a policy that will result in the tannery producing at the socially efficient level. If the policy were implemented, the town as a whole would have to pay a cost of \$400 (thousand) associated with administration and enforcement. This cost would be paid immediately (i.e. in year 0). The annual net benefit of having this policy in place would be the efficiency gain calculated in part a), which would be received each year starting in year 1. (If you were not able to calculate this in the previous part, you can assume a value for it for this part as long as you explicitly state your assumption.) The town council requires all cost-benefit analyses to use a 77-year time horizon and a discount rate of 3.3%. Calculate the net present value of implementing the policy. Based on this calculation, would you recommend that the town put the policy in place? Explain. (You can use formulas from class without deriving them for this question, but be careful to write them down clearly and completely, and to show your calculations.)

answer:

$$\delta = 1/1.033 = 0.9681$$

$$NPV = -C_0 + \frac{\delta - \delta^{78}}{1 - \delta} NB_{1-77}$$

$$= -400 + 27.8155*(21.6)$$

$$= \$200.82 \text{ (thousand).}$$

Since $NPV > 0$, the project would be an improvement on the status quo. [I took a point or two off for imprecise statements about why a positive net present value should or should not lead to recommending the policy.] It is fine to say that we would want to recommend it to the town on this basis, or also that we might want to do some sensitivity analysis.

c) [8 points] The town council is uncomfortable with the cost estimate of \$400 (thousand) and decides to investigate further. After this investigation, the expert consensus is that there is a 40% chance that the cost will indeed be \$400 (thousand), but a 25% chance that it will be \$220 (thousand) and a 35% chance that it will be \$620 (thousand). In addition, the town council decides that an infinite planning horizon should be adopted for the evaluation of this policy, and is considering adopting a different discount rate as well. Find the discount rate at which the town council would be indifferent between implementing the policy or not.

answer:

$$EV(C_0) = 0.25*220 + 0.4*400 + 0.35*620 = \$432 \text{ (thousand).}$$

$$NPV = -EV(C_0) + NB/r$$

$$= -432 + 21.6/r$$

For the town council to be indifferent between undertaking the policy and not, it must be that it is exactly as worthwhile in expected net present value terms to do the policy as it is to refrain from doing the policy and hence retain the status quo. The value of the status quo is zero, because without the policy, the cost will not be borne and the efficiency loss will not be corrected, and both of these are measured relative to doing nothing. (Since the decision is being made by a collective representative body rather than a private individual, it is sensible to assume that the start-up cost will have to be paid for through some kind of taxation or borrowing, rather than through drawing down some existing stock of funds. Therefore, the proper outside option is truly doing nothing – with a benchmark value of zero – as opposed to some alternative investment.)

So, for indifference,

$$NPV = 0$$

$$21.6/r = 432$$

$$r = 21.6/432 = 0.05 = 5\%.$$

(This is known as the “internal rate of return” of the policy.)

2. [2 parts, 28 points total] Consider a non-renewable and non-recyclable natural resource. Society only places value on this resource for the present period and the immediately following period (called periods 1 and 2 respectively), and there will be no exploration for this resource over this time. Marginal benefits to society are represented by the inverse demand function $P_i = 10 - 0.2Q_i$ for each period $i = 1, 2$ (where Q_i is the quantity of the resource that would be extracted/consumed in period i at price P_i dollars per unit) and marginal extraction costs are \$2.00 per unit in each period. The stock of the resource is fixed at 60 units.

a) [12 points] Calculate the dynamically efficient allocation of the resource across the two periods. Do this first with a discount rate of 6% and then again with a discount rate of 12%. Does the change in the efficient allocation with the change in discount rate make sense? Briefly explain why or why not.

answer:

The present-value marginal net benefit functions are:

$$PV(MNB_1) = 10 - 0.2Q_1 - 2 = 8 - 0.2Q_1$$

$$PV(MNB_2) = 1/(1+r)^*(8 - 0.2Q_2) = 1/(1+r)^*[8 - 0.2(60 - Q_1)].$$

(For period 2, the appropriate scarcity constraint $-Q_1 + Q_2 = 60 -$ has been rearranged and substituted in.)

For efficiency, $PV(MNB_1) = PV(MNB_2)$

$$r = 6\%$$

$$8 - 0.2Q_1 = 1/(1.06)^*[8 - 0.2(60 - Q_1)]$$

$$Q_1^{**} = 30.29, Q_2^{**} = 60 - 30.29 = 29.71.$$

$$r = 12\%$$

$$8 - 0.2Q_1 = 1/(1.12)^*[8 - 0.2(60 - Q_1)]$$

$$Q_1^{**} = 30.57, Q_2^{**} = 60 - 30.57 = 29.43.$$

With the 12% discount rate, the efficient allocation is slightly more skewed towards first-period consumption (i.e. $30.57 > 30.29$). This makes sense because, the higher the discount rate, the less weight the planners in the first period are putting on the value of consumption in the second period. It follows that, when relatively more importance is placed on the present (i.e. when $r = 12\%$), efficiency would require that more consumption be undertaken in the present.

b) [16 points] Now consider a more general case in which the non-renewable resource is a fossil fuel. Besides the issue of scarcity, there is the additional problem that the use of the resource imposes a negative externality in the present and the future (due to the associated pollution, which can cause immediate as well as long-lasting environmental damage). It has been suggested that consumers of the resource be charged a per-unit tax on each unit used. Assess this suggested policy in terms of its potential ability to achieve efficiency. Also, mention at least one potential shortcoming of the suggested policy, and a way in which the policy could be improved. Explain.

answer:

One can think of this essentially as the question of whether gasoline taxes should be higher. In a sense, this is getting at the heart and soul of the whole course. I don't care a whole lot about how good you get at doing little math problems for their own sake. I care a lot more about your ability to draw together the tools and reasoning we've been developing in order to think concretely about an extremely policy-relevant question. It's a very open-ended question, so I'll just try to touch on some main points here.

We get lots of benefits from using fossil fuels, but they are accompanied by (at least) two big drawbacks. First, their use causes environmental damage. Second, their use in the present means that less will be available in the future. The remedy to both of these problems, in a broad qualitative sense, is to limit our consumption, and seek a balance between the benefits and drawbacks of using fossil fuels. Because both of these drawbacks are social in nature (i.e. consumption decisions are made at the individual level but the associated costs are borne by society broadly), there is good reason to think that unrestrained markets will not lead to the "right" (i.e. socially efficient, the economist's usual metric) balance. We demonstrated separately for negative externalities and scarce non-renewable resources that the market will lead to too much consumption, so this should also be true with both of these factors at play.

We also demonstrated, for negative externalities, that the market over-production relative to efficiency could be corrected by imposing a tax, which would make the good more expensive and hence lead to less consumption of it. One can easily extend this logic to the scarce resource case. A tax administered on consumers is represented by an inward shift of the demand curve, with the taxed equilibrium occurring at a quantity below the market quantity, where the shifted demand curve crosses the marginal cost curve. (Many people analyzed the situation as a tax on producers, which would shift the supply curve up. This is not what the question asked for, but was not a serious mistake, as the two policy designs are effectively equivalent. However, this sometimes led to some more serious mistakes, discussed further below.) The simple answer is that the situation calls for a reduction in consumption, and that such a reduction can be very effectively achieved by imposing a tax. The tax policy thus has high potential ability to achieve efficiency.

To get a little more specific, though, we should be a bit more careful about what is required for efficiency. If the externality is much more severe than the scarcity of the

resource, analyzing the externality in isolation would lead us to recommend a much higher tax rate than if the scarcity were analyzed in isolation. So the specific level of the tax will have to be chosen to appropriately weigh these two concerns, and in general a holistic analysis of the situation would lead us to recommend a tax rate somewhere between the two extreme tax rates from the isolated analyses. If the externality is especially severe, it might even be efficient to set the tax rate so high that the resource will not end up being fully exhausted over the entire planning horizon.

Moreover, it is important to consider the time dimension with the environmental damage. As was discussed concerning climate change, one type of pollutant that is emitted as a result of burning fossil fuels is greenhouse gases, which can become concentrated in the upper atmosphere and contribute to intensifying the greenhouse effect. Because it is concentrations (or, in other words, stocks of accumulated emissions) that lead to the ultimate damages, emissions at a given point in time can contribute to damages over many periods in the future. In the simplified two-period set-up, we would want to think of society in period 1 suffering damage just related to emissions in period 1, but of society in period 2 suffering damage related to emissions in periods 1 and 2. This suggests that consumption of the resource in period 1 should probably be taxed at a higher rate than consumption in period 2, since the associated emissions do damage in two periods, whereas emissions in period 2 just do damage in period 2. (The same basic logic would carry through with longer and unbounded time horizons, so this is not just a curiosity of the model's simplifications. The important point is that emissions in future periods cannot cause damage in periods that have already passed, but emissions in the present can most certainly cause damage in future periods.) On the other hand, discounting the future implies that we in the present place less weight on benefits enjoyed as well as costs borne in the future. So the correct tax rates would need to appropriately account for both the concentration of emissions and discounting.

In thinking about shortcomings and possible improvements to the policy, I was trying to push you away from focusing on efficiency exclusively. One big problem with the policy is that it will probably hit consumers of the resource hard. Demand for fossil fuels is typically modeled as being quite inelastic, because they play an integral function in our daily lives and there are limited feasible substitution possibilities. This implies that the effective burden of taxation will be large and mostly borne by consumers. (It will be totally borne by consumers in the simple example even though the demand curve is not especially inelastic over most of its domain, because the supply curve is perfectly elastic. Again, the very simple model makes a lot of unrealistic assumptions, but can still manage to capture many important characteristics.) This is a concern because of the hardship it would impose on many consumers (and perhaps some income groups much more than others), and because of the political difficulties this hardship would create in getting such a tax policy passed into law.

Another shortcoming of the policy (or at least our analysis of it) is that we have not considered other markets. Besides consuming fossil fuels ourselves, they are also important intermediate inputs into the production of other things we consume and industries in which we work. Failing to take a general equilibrium perspective in setting

the tax rates could lead to unforeseen inefficiencies in other markets. In the same vein, one might think that setting the tax rates focusing on pollution and scarcity of the resource alone might fail to give adequate concern for national security issues (a full analysis of which would require an expanded model to analyze trade relations), though here at least we would have to argue that a tax policy would move consumption in the right direction.

The obvious way in which the tax policy could be improved would be to specify things that the tax revenue could be used for. Remember, there is nothing inherently good about the government just accumulating a lot of tax revenue, but there are potential efficiency gains to be had if the government uses tax revenue for things that private consumers would not otherwise do. For example, the government could use taxes collected during the first period to invest in knowledge and technologies related to cleaning up or adapting to damages in the second period. Alternatively, the revenue could be used to fund exploration for new reserves of the resource in question. While the first of these examples is focused on alleviating damages and the second on alleviating scarcity, a third could simultaneously address both: the tax revenue could be invested in the development of feasible renewable substitutes to the resource in question. The revenue could also be used for other worthwhile things unrelated to the resource, such as reducing more distortionary taxes (like income taxes) or investing in research and development more generally.

A more subtle way in which the tax policy could be improved would be a broadening of its application. An effective approach to climate change will never entail a simple increase in the gasoline tax, since the tax would have to be many times its current level to achieve an amount of carbon emissions approaching the efficient magnitude. Leaning this heavily on a single commodity tax to deal with a pollutant as widespread as carbon would entail extraordinarily large welfare costs. These could be partially mitigated by broadening the tax base to include all commodities and behaviors related to carbon emissions. The parallel in the simple set-up of the question would be to tax the use of other fossil fuels and related resources in addition to the one we have been focusing on. (As several people pointed out, taxing just this one resource alone could also have the perverse effect of increasing pollution, if there is enough induced substitution to other resources that pollute more. If we had to pay \$100/gallon for gas, it probably wouldn't be long before someone figured out how to make cars run on coal instead, which wouldn't really help us out with reducing pollution.) Finally, a few people mentioned that one way to overcome the political difficulties of implementing a tax policy such as the one suggested could be to use some of the projected revenue to run an education campaign on the merits of the policy. This type of campaign is sometimes referred to as "moral suasion", and can perhaps border on propaganda. The point, though, is that individuals might be more willing to make sacrifices for future generations if they understand the benefits these generations stand to gain and are convinced that the policy has been designed in order to limit the necessary sacrifice as much as possible.

There were a few recurring suggestions that are worth pointing out for their flaws. Some people suggested that producers instead of consumers should be taxed or that the

government should somehow “shift the burden” of taxation onto producers rather than consumers. This misses the equivalence in effective burdens that holds regardless of which side of the market bears the administrative burden. Further, I would argue that the political gain to be had over past decades from talking loudly of “making polluters pay” and going after firms more generally has largely worn off as voters have learned that prices will rise with regulation regardless of the intended target. Some suggested that the tax policy could be improved by removing it and replacing it with a tradable permits system. I don’t see how this could help, since meeting a given target through price-side controls is fully equivalent to meeting it through quantity-side controls (in the absence of uncertainty). Further, I would argue that a permit system would be more administratively costly in this case, because fossil fuel use is so widespread and there would hence be billions of people wanting to buy and sell relatively small quantities of permits for their daily use. Finally, some people thought that the policy would not be great because it would be very difficult to know the precise levels at which to set the tax (or quantity limit). I don’t think that’s a whole lot different for most other potential policies, including alternatives to the tax policy in question, once we leave the textbook examples. I would also argue that, while claiming to have found a policy that could perfectly achieve efficiency in the real world is complete hubris, economists are quite capable of designing policies that are surely a large improvement, both on the status quo and on more traditional and “command-and-control” forms of regulation. (Also, some people suggested that the valuation techniques we discussed could be used to help find the right level of the tax in a cost-benefit framework, which is not really what the question was trying to get at, but which was a valid enough point.)

As usual, this is much more information than I require for full credit. Roughly, half of the points was allotted to the first part of the question, on how the tax could functionally address the two issues. Of this, only a couple were for thinking about how tax rate(s) could be chosen to balance objectives across time and specific concerns. The other half of the points was for suggesting a shortcoming and an improvement, with a few reserved for at least some exposition of your rationale. All well-argued responses were eligible for full credit.

3. [3 parts, 24 points total] The biological relationship between the growth of a given fish population and the population size can be expressed as $g = 600x - 10x^2$, where g is the net addition to the stock in number of fish and x is the size of the stock in thousands of fish.

a) [8 points] Find all of the biological equilibria of this fishery, and state which of these are stable and which are unstable. Find the maximum sustainable yield for this fishery.

answer:

The two biological equilibria are where $g = 0$, i.e. $x = 0$ and where $600x = 10x^2$, or $x = 60$. The latter (60) is stable (and is called the carrying capacity), and the former (0) is unstable.

By taking the derivative of $g = 600x - 10x^2$ with respect to x and setting it to zero, we will be able to solve for x^* , the population level corresponding to the maximum sustainable yield (and to the highest point on the graph of the g function).

$$g' = 600 - 20x = 0 \rightarrow 20x = 600 \rightarrow x^* = 30 \text{ thousand fish.}$$

This is NOT the *MSY* itself, nor is it a biological equilibrium! The maximum sustainable yield will be the number of fish by which the population is growing at population x^* :

$$MSY = g^* = 600x^* - 10(x^*)^2 = 600*30 - 10*900 = 9,000 \text{ fish.}$$

b) [8 points] Suppose that this population is being harvested at a fixed rate of 3,500 fish per period. Starting from an initial population of $x_0 = 47$ (thousand) fish at the end of period 0, calculate the population levels at the end of the next two periods (i.e. calculate x_1 and x_2). You must show all of your calculations to get full credit. You can round to one decimal place.

answer:

$$\begin{aligned}x_1 &= x_0 + (g(x_0) - h_0)/1000 \\&= 47 + ([600*47 - 10(47)^2] - 3500)/1000 \\&= 49.6\end{aligned}$$

$$\begin{aligned}x_2 &= x_1 + (g(x_1) - h_1)/1000 \\&= 49.6 + ([600*49.6 - 10(49.6)^2] - 3500)/1000 \\&= 51.3.\end{aligned}$$

c) [8 points] The bionomic equilibrium total revenue function associated with this population is given by $TR = 230e - 0.6e^2$, where e is fishing effort (measured in weeks per year at sea). The marginal cost of fishing effort is constant at \$200 per unit of effort expended, and there are no fixed costs associated with fishing. Compute the effort level that would arise if there were open access to this fishing area. Compute the efficient effort level.

answer:

Total costs are given by $C = 200e$, by the assumptions stated in the question.

With open access, the fishery will be exploited to the point that total profits are driven to zero:

$$\begin{aligned}\pi &= R - C = 0 \rightarrow R = C \\ 230e - 0.6e^2 &= 200e \\ 30e &= 0.6e^2 \\ 30 &= 0.6e \\ e^{OA} &= 30/0.6 = 50.\end{aligned}$$

$$MR = TR' = 230 - 2*(0.6)*e = 230 - 1.2e.$$

The socially efficient effort level is that which maximizes profits in this case, so the usual profit-maximizing principal can be applied:

$$\begin{aligned}MR &= MC \\ 230 - 1.2e &= 200 \\ 1.2e &= 30 \\ e^{eff} &= 30/1.2 = 25.\end{aligned}$$

4. [2 parts, 20 points total] Consider an economy with three firms that emit an environmentally harmful uniformly mixed fund pollutant as a by-product of their production processes. These emissions are perfectly and costlessly monitored by the government. The marginal cost relations faced by each firm for abating a given amount are $MC_1 = 8 + 4q_1$, $MC_2 = 6 + 3q_2$ and $MC_3 = 4 + 2q_3$ (in dollars) where q_1 , q_2 and q_3 are the units of reduction undertaken by firm 1, firm 2 and firm 3 respectively. Each firm would have baseline emissions of 147 units in the absence of any regulation.

a) [12 points] Suppose that the government sets an aggregate emissions target for this pollutant of 252 units. It introduces a uniform emissions fee policy, and initially sets the fee level at \$156/unit. In order to meet the aggregate emissions target, will the government need to decrease the fee level, increase the fee level or keep the fee at this level? Explain your answer without calculating the appropriate fee level. State explicitly any assumptions you are making about how these firms behave.

answer:

We assume that firms are individual cost minimizers (and implicitly we always assume that they are perfectly competitive). [This was worth 2 of the 12 points.] They do not care about cost effectiveness at the economy-wide level, but rather choose abatement so that the sum of their total private abatement costs and total private fee payments is minimized. They do this by setting their own MC of abatement to the fee level (by our argument from class).

$$MC_1 = 8 + 4q_1 = 156 \rightarrow q_1^m = 37$$

$$MC_2 = 6 + 3q_2 = 156 \rightarrow q_2^m = 50$$

$$MC_3 = 4 + 2q_3 = 156 \rightarrow q_3^m = 76.$$

So economy-wide abatement will be $37 + 50 + 76 = 163$ with the fee at this level. What is the aggregate abatement target? By the formula that should be familiar by now – emissions equals baseline emissions minus abatement – $R = (147 \cdot 3) - 252 = 189$. That is, the aggregate abatement level that results from the cost-minimizing behavior of the three firms individually is too low relative to the target. The fee thus needs to be increased, because this will cause each of the firms to adjust its abatement level upwards so as to avoid the then-higher fee on the next few units. (The higher the fee, the sharper is the individual incentive to bear costs in the form of abatement rather than fees.) [A point or two was taken away for saying that the fee would have to be changed but not explaining the mechanism by which this would help.]

b) [8 points] Now suppose that the government removes the fee policy, and instead implements a tradable permits system. In addition, the aggregate emissions target is lowered to 200 units. All of the permits are auctioned, and at the end of the auction, the cost-effective abatement allocation of (55, 74, 112) is achieved. Then firm 3 decides to exit the economy and cease all emissions. By previous arrangement, firm 1 is given three fifths of firm 3's permits at no cost, and firm 2 is given the remaining two fifths of firm 3's permits at no cost. After firm 3 exits and its permits are thus distributed, firm 1 and firm 2 are free to exchange permits amongst themselves. The aggregate emissions target remains fixed at 200 units. Which firm will buy permits, and how many will it buy?

answer:

The cost-effective abatement allocation given in this part corresponds to a permit allocation of [92, 73, 35] (found by subtracting a firm's abatement from its baseline emissions, which are 147 for each firm, from the main set-up for the problem). Firm 3's 35 permits will be distributed as per the arrangement described in this part, so that the two-firm permit allocation will be [113, 87], i.e. $92 + 3/5 \cdot 35$ for firm 1 and $73 + 2/5 \cdot 35$ for firm 2.

The general lesson of tradable permits modeling is that firms should have private incentives to bargain to the cost-effective allocation from any initial allocation. In this framework, we can treat [113, 87] as the initial allocation. What is the cost-effective allocation? We find that in terms of abatement by applying the equimarginal principle and the reduction constraint. What is the aggregate reduction target? We need to apply that familiar formula again, but being especially careful to note that firm 3 has exited along with its baseline emissions and that the aggregate emissions target is now 200.

$$R = (147 \cdot 2) - 200 = 94$$

$$8 + 4q_1 = 6 + 3(94 - q_1) \text{ for cost effectiveness...}$$

$$q_1^{ce} = 40, q_2^{ce} = 54.$$

This cost-effective abatement allocation of (40, 54) corresponds with an emissions allocation of [107, 93], again using baseline emissions of 147 for each firm and the familiar formula.

Now we can see clearly that firm 1 has six permits too many in the initial allocation compared to what is cost effective (i.e. 113 rather than 107). Symmetrically, firm 2 has six too few in the initial allocation compared to what is cost effective. So firm 2 will buy six permits from firm 1, and the cost effective allocation will be achieved (by our argument concerning private bargaining incentives from class).

BONUS QUESTION [6 points maximum – no penalty for guessing]: Returning to the third problem and assuming that the baseline harvest function takes the usual general form, calculate what the price per fish and the catchability coefficient must be. Also, find the efficient and open-access harvest levels.

answer:

The general form of the baseline harvest function we use is $h = cex$, where c is the catchability coefficient.

For bionomic equilibrium, $g - h = 0$

$$g = h$$

$$600x - 10x^2 = cex \text{ (growth function from set-up to third problem)}$$

$$600 - 10x = ce \text{ (divided through by } x)$$

$$10x = 600 - ce$$

$$x = 60 - ce/10.$$

The revenue function in terms of effort and the parameters only is found by substituting this expression for x into the baseline harvest function and multiplying by the price, which we represent with p .

$$TR = ph$$

$$= pce(60 - ce/10)$$

$$= 60pce - pc^2e^2/10.$$

Going back to part c) of the third problem, the revenue function is given there as $TR = 230e - 0.6e^2$. So we need to find the values for the parameters p and c that make the coefficients in the general form of the revenue function match up with those in the specific form. This entails solving a system of two equations in two unknowns:

$$60pc = 230 \text{ and } pc^2/10 = 0.6.$$

Solving this system should give $c = 36/23$ and $p = 529/216$.

To find the harvest levels, we need to use the effort levels found in the solutions to part c) of the third problem, the value for c just found and the expression from above describing the relationship between x and e in bionomic equilibrium. Substituting all of this information into the baseline harvest function,

$$h^{OA} = (36/23)*50*[60 - (36/23)*50/10] = 4083.176$$

and

$$h^{eff} = (36/23)*25*[60 - (36/23)*25/10] = 2194.707.$$