EC371 – Environmental Economics, Fall 2010, Boston University

Instructor: Jeremy Smith

Second Mid-term Exam

Wednesday, December 1, 2010

This is a 46-minute exam, but you will have 51 minutes to complete it. There is a total of 46 points allocated across two questions. In addition, there is one bonus question at the end. Use the number of points allocated to each part as a rough guide to how long to spend on that part. I recommend that you use one minute per point *at most* until you have gotten through each question, then use your extra time to revisit parts you may have missed the first time through and to check your work.

Please read the questions carefully and write your answers in the space provided. You can use the backs of the sheets for scrap paper, but to get full credit you must show all relevant work in the space provided.

Please follow my instructions at all times.

Concentrate and think carefully, but try to relax too!

Student Number: Solutions

(Please do not include your name.)

1. [28 points total, 3 parts] Consider an economy with two firms that emit an environmentally harmful uniformly mixed fund pollutant as a by-product of their production processes. These emissions are perfectly and costlessly monitored by the government. Suppose the government decides that an aggregate abatement target of 147 units must be met. This corresponds to an aggregate emissions target of 150 units. The marginal cost relations faced by each firm for abating a given amount are $MC_1 = 6q_1$ and $MC_2 = 3q_2$ (in dollars) where q_1 and q_2 are the units of abatement undertaken by firm 1 and firm 2 respectively.

a) [10 points] Calculate the cost-effective allocation of individual abatement requirements that satisfies the total abatement requirement. Assuming that this cost-effective abatement allocation corresponds with firm 1 emitting 128 units, verify that firm 2's baseline pre-regulation emissions are 120 units.

answer:

Apply the equimarginal principle ($MC_1 = MC_2$) and the reduction constraint ($q_1 + q_2 = 147$): [6 points]

$6q_1 = 3(147 - q_1)$
$q_1 = 49$, $q_2 = 147 - 49 = 98$,

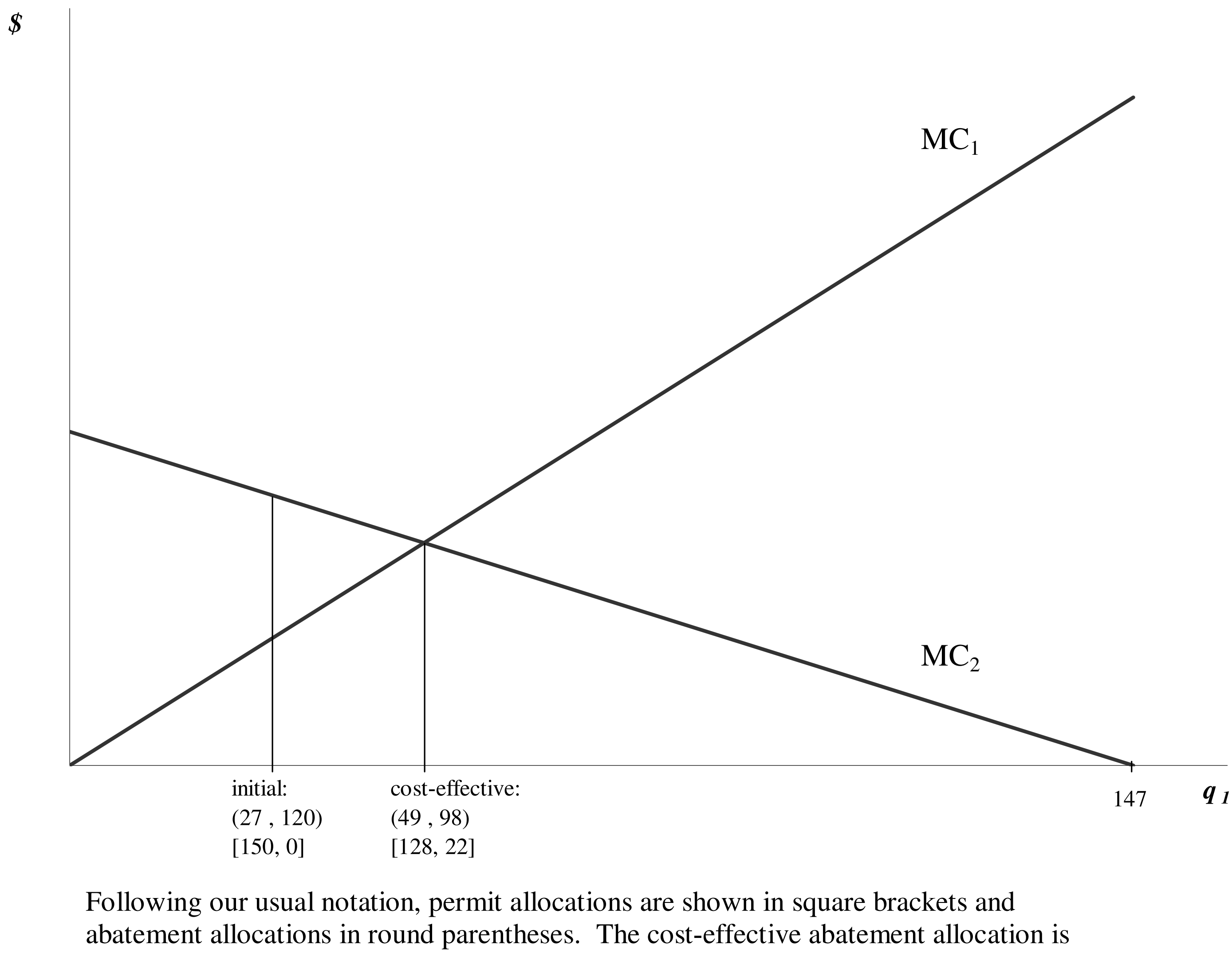
so the cost-effective abatement allocation is (49, 98). [1 point]

To find firm 2's baseline emissions, there are a couple of ways to proceed, but they all require at least some use of the general relationship that emissions equal baseline emissions minus abatement. One way is to note that economy-wide baseline emissions are 297 (implied by the aggregate abatement target of 147 corresponding to an aggregate emissions target of 150, as given) and firm 1's baseline emissions are 177 (implied by firm 1's cost-effective abatement being 49, as found above, and its corresponding emissions being 128, as given). If total baseline emissions are 297 and 177 of these are from firm 1, then firm 2's baseline emissions must be the remaining 297 – 177 = 120 units, as required. (Another way is to note that firm 2's cost-effective emissions must be 22, since firm 1's emissions plus firm 2's emissions must add up to the emissions target of 150, and firm 1's cost-effective emissions are given as 128. Then firm 2's baseline emissions can be found as its abatement plus emissions at the cost-effective allocation: 98 + 22 = 120, again as required. There are probably other variations, any of which are fine as long as they are based on valid calculations and logic.) [3 points]

This last part just required churning through some logical connections. There's nothing too deep or intuitive about it, but it does lay the groundwork for the next part.

b) [10 points] Draw a single graph showing the marginal abatement cost curve for each firm, accounting for the fact that the aggregate reduction requirement must always be met. (Be sure to label your axes, horizontal intercept and curves. The scale does not have to be perfect, but should be approximate.) Suppose that the government implements a tradable permits system. It prints 150 permits and gives them all away to firm 1. Label the point on the horizontal axis of your graph that corresponds to the cost-effective allocation, and express this allocation both in terms of the abatement levels of each firm and the number of permits held by each firm. Do the same for the point corresponding to the initial allocation. Imagine that, once all incentives amongst the firms to exchange permits have been exhausted, the government prints one additional permit. What price would each firm be willing to pay for this imaginary extra permit?

answer:



Following our usual notation, permit allocations are shown in square brackets and abatement allocations in round parentheses. The cost-effective abatement allocation is from part a), and its conversion to a cost-effective permit/emissions allocation is based on the discussion of firm 2's baseline emissions in the solutions to that part. For the initial allocation, start with the given information that all 150 permits are handed to firm 1. This implies that there are zero permits leftover for firm 2, and so the initial permit allocation is [150 , 0]. This is converted to an abatement allocation my using information on the baseline emissions for each firm (discussed in the previous part) and the usual relationship: 177 – 150 = 127 units abated by firm 1 and 120 – 0 = 120 units abated by firm 2.

[7 points. Points deducted for: missing or incorrect labels; incorrectly-shaped curves; incorrect placement of allocations; incorrect calculation of allocations; and grossly incorrect scale (MC_2 should be at least a bit less steep in absolute value than MC_1). Up to 4 points in partial credit for calculating all allocations correctly.]

(Just as a check, note that both abatement allocations have components adding up to 147 and both permit allocations have components adding up to 150, as required. Note that it's not possible for firm 2 to abate any more than 120 units and it's not possible for firm 1 to abate any more than 177 units. This does not affect the analysis and it is not necessary to include this information on the graph in the form of vertical lines; doing so at the correct abatement levels was not penalized, but doing so at incorrect levels was penalized.)

The bit about the imaginary extra permit is a way of thinking about the "market price" for permits at the cost-effective allocation, and is also what I was trying to get at by asking about the "terminal price" in the practice problem on this topic. The point is that, once the firms have exchanged permits and bargained their way to the cost-effective allocation (which, as we've argued in class, they have private incentives to do), the "market" for permits essentially shuts down, because there are no buyers and sellers willing to trade with one another. But this just means that there is no more room to bargain over a transaction price for the existing permits, not that the firms wouldn't be willing to pay a positive price for a permit if someone else came along and was willing to sell one at an agreeable price. The firms would place value on having an additional permit because this would allow the buying firm to save money by reducing its abatement by one unit. How much would it save? At the cost-effective allocation, the marginal abatement costs of the firms are equalized (by definition), and the level at which they are equalized represents how much each could save by lowering abatement by one unit. So this height gives the maximum price that either firm would be willing to pay for an additional permit. That height is found by plugging either firm's cost-effective abatement level into its marginal abatement cost function: 6*49 = 3*98 = $294 = p^*, the market price for permits at the cost-effective allocation. It wasn't necessary to show this on the graph. [3 points]

c) [8 points] The economy-wide marginal damage costs from emissions of the pollutant in question are given by $MD = 0.0384Z^2 - 2Z - 6$, where Z is the aggregate level of emissions of the pollutant at any point in time. The economy-wide marginal control cost curve is given by $MC = 2Q$, where Q is the aggregate amount of emissions abated at any point in time. Find the efficient level of aggregate emissions of the pollutant in this economy. Calculate how much higher economy-wide total abatement costs would be if the permit policy from the previous part were changed to meet the efficient target.

answer:

Economy-wide baseline emissions in this case are 297 (as argued in the solution to the first part), so we can write the general relationship between aggregate abatement and aggregate emissions as $Q = 297 - Z$.

Efficiency is obtained where total overall damage plus control costs are minimized, which is where MD=MC as argued in class.

$MD = MC$ [2 points]
$0.0384Z^2 - 2Z - 6 = 2Q$
$0.0384Z^2 - 2Z - 6 = 2(297 - Z)$ (substituting in the expression from above) [2 points]
$0.0384Z^2 - 600 = 0$
$Z^2 = 600/0.0384$
$Z^{**} = \sqrt{15625} = 125.$ [1 point]

So the emissions target of 150 from the previous parts is not ambitious enough from an efficiency perspective. (As was discussed in class, there's no reason for the target set by one level of policy to need to correspond to the efficient target in order for us to undertake cost-effectiveness analysis. The cost-effective abatement allocation found in the first part thus turns out to be a cost-effective way of achieving an inefficient target, but it's cost effective nonetheless. Advice on how to reach a given aggregate target at least cost is still valuable from a policy perspective whether or not the given target happens to be the one that economists would favor.)

Total abatement costs can be calculated separately for the two firms using the individual marginal abatement cost functions from the beginning of the problem, first for the cost-effective allocation associated with the earlier target of 150 units of emissions, then again for the cost-effective allocation associated with a new target adjusted to achieve efficient emissions of 125 units (which would have to be found by applying the equimarginal principal again). The quicker way is to use the economy-wide marginal control cost curve (remember, we use "control", "abatement", "reduction" and the like synonymously). Emissions of 150 correspond to control of 147, and emissions of 125 correspond to control of 297 – 125 = 172. So we need to calculate TAC(147) and TAC(172), which are just the areas of triangles under $MC = 2Q$ up to the given aggregate control levels. These should come to \$21,609 and \$29,584, for an increase in economy-wide total abatement costs of 29584 – 21609 = \$7,975 after moving the emissions target to the efficient level of 125. (There would be no social loss from meeting the target cost ineffectively, since the permit system is a cost-effective policy instrument.) [3 points]

Does it seem a little strange that costs increase when moving from an inefficient target to the efficient target? Remember that abatement costs are only part of the efficiency calculation, with environmental damage being the other part. The original emissions target of 150 is inefficient because it is forcing environmental costs to be too high relative to control costs. Overall costs can be decreased by accepting moderate increases in control costs in return for larger drops in damage costs by increasing abatement (and hence decreasing emissions); this trade-off remains favorable until the intersection of *MD* and *MC* is reached, defining the efficient level of emissions.

2. [18 points total, 2 parts] Consider a non-renewable and non-recyclable natural resource that currently has no substitutes. Society only places value on this resource for the present period and the immediately following period (called periods 1 and 2 respectively), and there will be no exploration for this resource over this time. Marginal benefits to society are represented by the inverse demand function $P_i = 10 - 0.2Q_i$ for each period $i = 1, 2$ (where Q_i is the quantity of the resource that would be extracted/ consumed in period i at price P_i dollars per unit) and marginal extraction costs are $2.00 per unit in each period. The stock of the resource is fixed at 90 units, but this is divided into two parts: 60 units are available for extraction with current practices, while the remaining 30 units are geologically isolated and cannot be extracted with existing technologies.

a) [10 points] Assuming that the last 30 units will never be obtainable, calculate the dynamically efficient allocation across the two periods of the 60 units that can be feasibly extracted. Do this first with a discount rate of 6% and then again with a discount rate of 12%. Does the change in the efficient allocation with the change in discount rate make sense? Briefly explain why or why not.

answer:

The present-value marginal net benefit functions are:

$PV(MNB_1) = 10 - 0.2Q_1 - 2 = 8 - 0.2Q_1$
$PV(MNB_2) = 1/(1 + r)*(8 - 0.2Q_2) = 1/(1 + r)*[8 - 0.2(60 - Q_1)]$.

(For period 2, the appropriate scarcity constraint – $Q_1 + Q_2 = 60$ – has been rearranged and substituted in.) [1 point]

For efficiency, $PV(MNB_1) = PV(MNB_2)$ [3 points]

<u>$r = 6\%$</u>
$8 - 0.2Q_1 = 1/(1.06)*[8 - 0.2(60 - Q_1)]$
$Q_1^{**} = 30.29$, $Q_2^{**} = 60 - 30.29 = 29.71$. [1 point]

<u>$r = 12\%$</u>
$8 - 0.2Q_1 = 1/(1.12)*[8 - 0.2(60 - Q_1)]$
$Q_1^{**} = 30.57$, $Q_2^{**} = 60 - 30.57 = 29.43$. [1 point]

With the 12% discount rate, the efficient allocation is slightly more skewed towards first-period consumption (i.e. 30.57 > 30.29). This makes sense because, the higher the discount rate, the less weight the planners in the first period are putting on the value of consumption in the second period. It follows that, when relatively more importance is placed on the present (i.e. when $r = 12\%$), efficiency would require that more consumption be undertaken in the present. [4 points]

b) [8 points] Suppose that, at the beginning of period 2, there is a technological innovation that makes the previously unattainable 30 units of the resource feasible to extract at the same fixed marginal extraction cost of $2.00 per unit. Sticking with a discount rate of 6%, find the efficient allocation of the resource in each of the following two cases: i. the government that sets the dynamic extraction plan at the beginning of the first period has no foresight and cannot anticipate the innovation at all; and ii. the government that sets the dynamic extraction plan at the beginning of the first period has perfect foresight and fully anticipates that the innovation will happen at the beginning of period 2.

answer:

This was a tough question. The big point that I was trying to demonstrate is that, in contrast to our baseline two-period model in which the stock of the resource is fixed and known with certainty, in reality the proven economically viable reserves of a given resource can increase over time as new sources are discovered and new extraction processes are developed. Moreover, such changes to feasible reserves can be very unpredictable and sporadic. How could a planner find the efficient consumption stream when the stock of the resource is an unpredictable moving target? There are some fairly ugly mathematical tools that allow us to incorporate general forms of uncertainty into the analysis, but without getting into them, the idea was to think about two extreme forms (namely, complete and none) that can be explored within the basic model. Of course, in reality, policy makers usually find themselves somewhere between these extremes.

However, there's another twist to the question, which in retrospect I think was too opaque and takes away from the broader point on uncertainty of the stock. When the extra units of the resource become available, the resource switches from being scarce to being abundant, i.e. with the full stock size of 90 units available, there is more of the resource than society wants in both periods. This can be seen by setting marginal benefits to marginal costs, which shows that society would want to consume 40 units in each period, which can be accomplished while still leaving 10 units in the ground.

<u>i. unanticipated</u>

If the planner at the beginning of period 1 has no foresight and hence no clue that the available stock will ever be anything besides 60 units, he will choose the efficient consumption plan that was found in part a). That is, the planner can do no better than behaving as though the stock size of 60 is known with certainty, since we have contrived the situation to be such that he has absolutely no other information. So, the planner will allow 30.29 units to be consumed in period 1 (using the discount rate of 6%, as specified in the question), saving the remainder for period 2. However, when period 2 arrives, the 29.71 previously attainable units are in the ground ready to be extracted, but now the 30 additional units are also available due to the innovation, for a total availability of 59.71. Society in period 2 is now capable of deviating from the consumption plan chosen in the previous period, but will it want to, and if so, will it choose to consume all 59.71 units? It can clearly make itself better off by increasing consumption above the level of 29.71

that it would have been stuck with in the absence of the innovation, but remember that the most it wants is 40, or more precisely, after 40 units have been extracted and consumed, the marginal benefit of additional units is not high enough to justify the cost it would take to extract them. So 19.71 units will remain in the ground and unconsumed. Note that this is purely from self-interest in period 2, and is not related to a desire to leave some of the resource for future periods. We are maintaining the assumption throughout that nobody cares about anything besides periods 1 and 2, or equivalently, that other periods simply don't exist.

So with a completely uninformed planner in period 1, society will consume 30.29 units in period 1 and 40 units in period 2 (and will not extract the remaining 19.71 units). In retrospect, we could improve on this situation by giving some of these remaining units to period 1, but it's obviously not feasible to go back in time to do this, and the planner at the beginning of period 1 would not want to consume more than 30.29 units ex ante because the best information he has indicates that doing so would impose too much cost due to scarcity on period 2.

<u>ii. fully anticipated</u>

If the planner at the beginning of period 1 is fully certain that the 30 extra units will become available by the beginning of period 2, her job is easy. She knows that allowing people in period 1 to consume as much of the resource as they want to (40) will still leave period 2 with 50 units available, which is more than people in period 2 will want to consume (40). So the planner can just allow the market to operate without any concern of imposing costs due to scarcity on future consumers. This will again lead to some units being left in the ground, but again this is purely an outcome of self-interest.

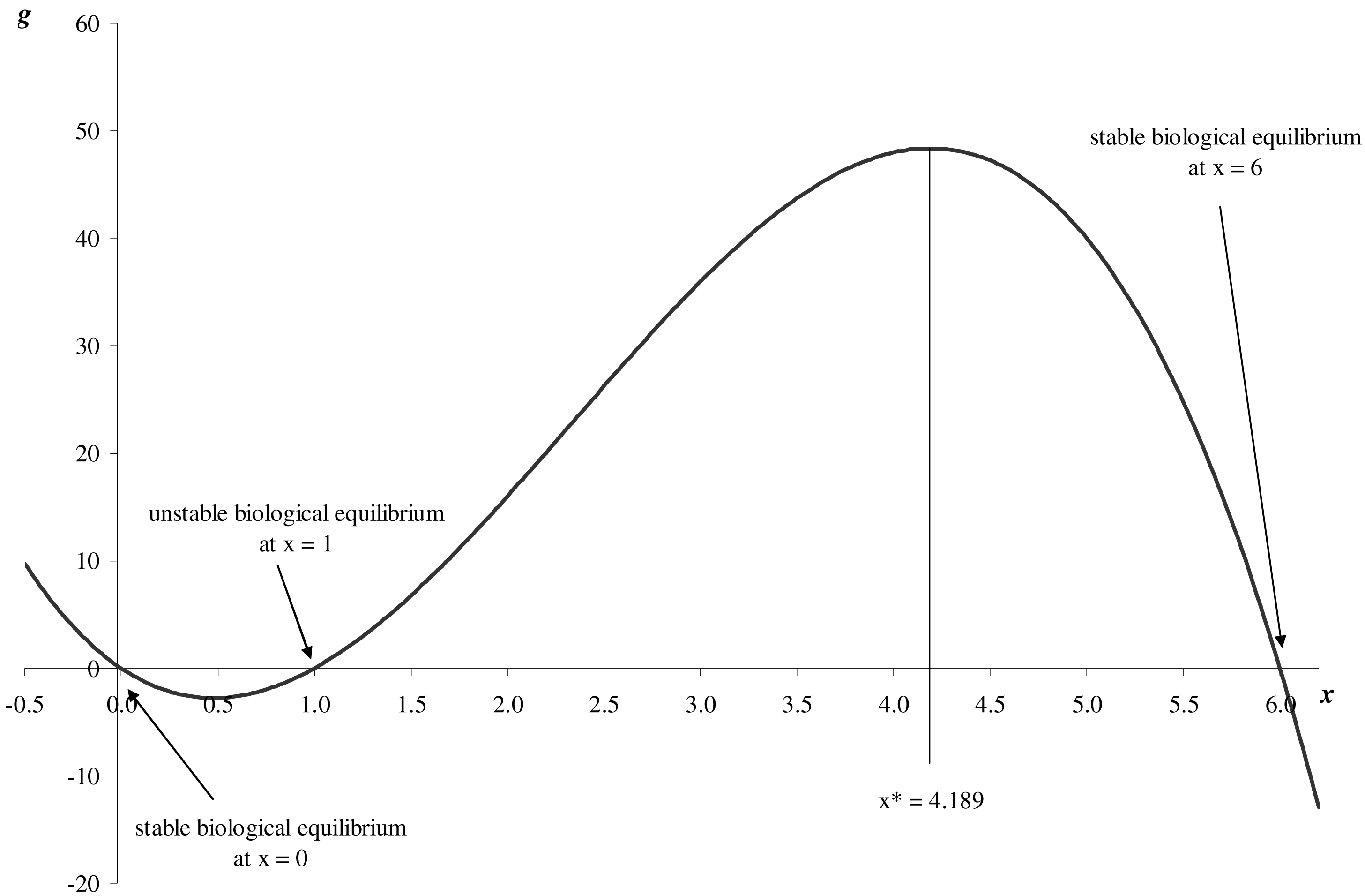
So with a fully-informed planner at the beginning of period 1, society will consume 40 units in period 1 and 40 units in period 2 (and will not extract the remaining 10 units at all). There is no regret in this case, i.e. there is no way ex post to hypothetically make either period better off by going back in time to adjust consumption levels.

An incorrect way to proceed with this part was to swap 90 for 60 in the equimarginal condition in part a) and solve for the quantities in each period. This does correctly deal with the uncertainty – recognizing that the planner would behave as though the total stock size is 90 with certainty – but it forces the entire stock of 90 to be exhausted, which we have argued is not desirable. The quantities come out to 44.85 and 45.15, which should seem a little strange because consumption in the second period is higher than in the first. Intuitively, if society is forced to over-consume the resource, the planner in period 1 will want to push more of the over-consumption to the future, where it hurts less due to discounting. If you check, you'll also find the marginal user cost to be negative at this allocation, which should be another red flag. This strange outcome is a result of not accounting for the abundance of the resource due to the innovation.

[4 points for each case; 2 points for each in partial credit for expressing good reasoning regarding uncertainty but not properly accounting for the abundance of the resource]

BONUS QUESTION [3 points maximum – no penalty for guessing]: The relationship between the growth of a given fish population and the population size can be expressed as $g = 14x^2 - 12x - 2x^3$, where g is the net natural addition to the stock in number of fish and x is the size of the stock in thousands of fish. Find the biological equilibria of this fishery and state whether each is stable or unstable, and find the stock size corresponding to the MSY.

answer:



This is an extension of the basic quadratic Schaefer model that is better able to account for natural extinctions and other phenomena. The unstable equilibrium at $x = 1$ is often referred to as the "minimum viable population": if shocked below this level, the population will not be able to breed effectively enough for births to replace deaths, and will slowly become extinct. Extinction – i.e. $x = 0$ – is now a stable biological equilibrium, since a positive shock that is not at least as large as the minimum viable population will again cause the population to slowly fall back to 0.

Finding the biological equilibria is accomplished, as usual, by setting $g = 0$ (i.e. looking for population levels that are associated with zero net natural growth and so remain constant over time). The growth function is a cubic, so has three roots. In this case, $x = 0$ is clearly one of the roots, but to find the other two requires dividing the equilibrium condition by x and solving the remaining quadratic (which can be factored cleanly with this particular function). Stability is determined, as in class, by considering the subsequent growth forces that would kick in after getting shocked away in either direction from each equilibrium. Finding x^*, the population level associated with the maximum sustainable yield (MSY), requires taking the derivative of the g function (an application of the power rule of differentiation, which was demonstrated for a quadratic in class and on the fourth set of practice problems and extends simply to the cubic case) and setting it to zero. This now requires solving another quadratic, which unfortunately isn't clean this time, so that the quadratic formula is needed. The larger of the two roots is x^*; the smaller root is the population level associated with the lowest point between the equilibria at $x = 0$ and $x = 1$, which does not have a meaningful interpretation.

[No graph necessary. No argument necessary for supporting claims of stability. There are seven individual pieces of information asked for: 1 point for writing down the appropriate conditions, showing some work and/or getting one piece of information correctly; 2 points for getting at least three pieces of information correctly; full points for getting at least five pieces of information correctly.]