

EC371 – Environmental Economics, Spring 2010, Boston University

Instructor: Jeremy Smith

Second Mid-term Exam

Thursday, April 15, 2010

This is a 50-minute exam, but you will have 60 minutes to complete it. There is a total of 50 points allocated across two questions. In addition, there is one bonus question at the end. Use the number of points allocated to each part as a rough guide to how long to spend on that part. I recommend that you use one minute per point *at most* until you have gotten through each question, then use your extra time to revisit parts you may have missed the first time through and to check your work.

Please read the questions carefully and write your answers in the space provided. You can use the backs of the sheets for scrap paper, but to get full credit you must show all relevant work in the space provided.

Please follow my instructions at all times.

Concentrate and think carefully, but try to relax too!

Student Number: Solutions

(Please do not include your name.)

1. [31 points total, 3 parts] Consider an economy with two firms that emit an environmentally harmful uniformly mixed fund pollutant as a by-product of their production processes. These emissions are perfectly and costlessly monitored by the government. The marginal cost relations faced by each firm for abating a given amount are $MC_1 = 15q_1$ and $MC_2 = 20q_2$ (in dollars) where q_1 and q_2 are the units of abatement undertaken by firm 1 and firm 2 respectively. Firm 1 has baseline emissions of 321 units in the absence of any regulation, while firm 2 has baseline emissions of 344 units in the absence of any regulation. The government has set an aggregate abatement target for this pollutant of 217 units.

a) [12 points] Calculate the cost-effective allocation of individual abatement requirements that meets the aggregate abatement target. Calculate the appropriate per-unit uniform emissions fee that the government would have to implement to achieve this allocation.

answer:

For cost effectiveness, $MC_1 = MC_2$

and, of course

$$q_1 + q_2 = 217$$

→

$$15q_1 = 20(217 - q_1)$$

$$q_1^{ce} = 124, q_2^{ce} = 217 - 124 = 93. \quad [8 \text{ points}]$$

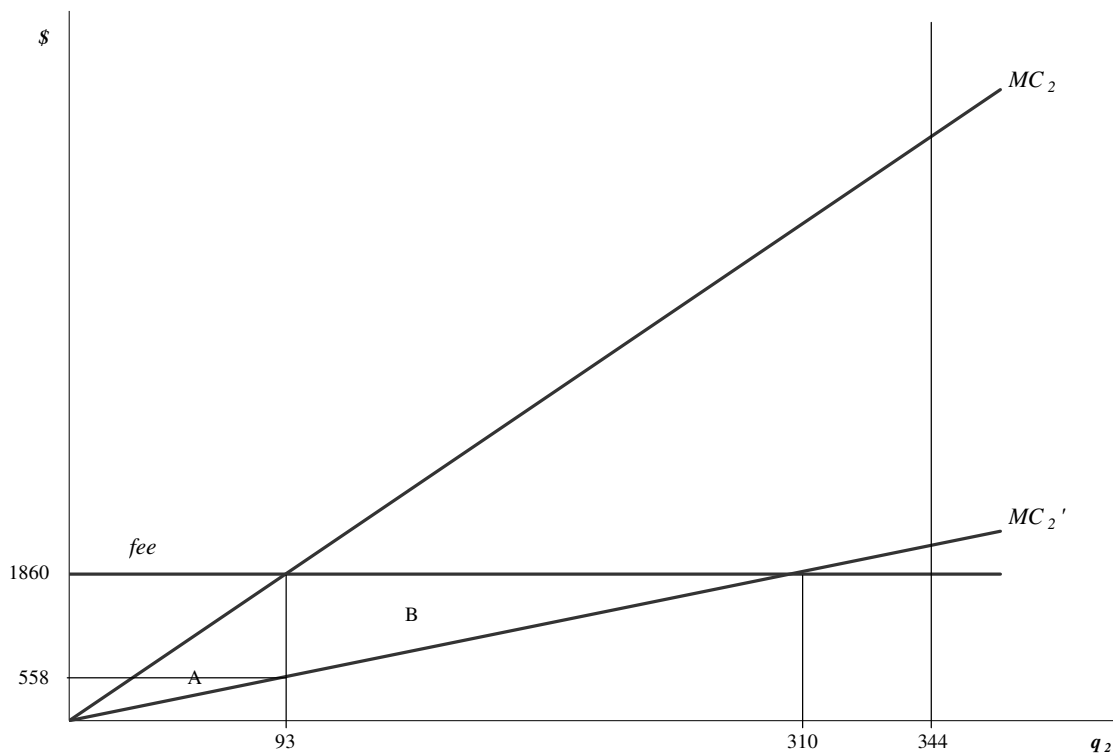
To find the appropriate fee:

$$f^{ce} = MC_1(q_1^{ce}) \text{ (or, equivalently, } f^{ce} = MC_2(q_2^{ce}))$$

$$f^{ce} = 15 * 124 = \$1,860/\text{unit}. \quad [4 \text{ points}]$$

b) [12 points] Suppose that the government decides to impose a uniform per-unit emissions fee at the level you found in the previous part. (If you were not able to complete the previous part, assume a value for the per-unit fee, and state this assumption explicitly.) Firm 2 is contemplating adopting a new abatement technology that would shift its marginal cost relation from $MC_2 = 20q_2$ to $MC_2' = 6q_2$. Firm 2 knows for certain that the government will not alter the fee level whether the technology is adopted or not, i.e. you can assume that the fee will stay fixed. What would firm 2's total cost savings in a given period be if it adopts the new technology? What is the total fee revenue that the government would collect in a given period with this policy in place, assuming that firm 2 adopts the new technology?

answer:



The graph is not necessary, but probably helps to see what's going on. With the old technology, firm 2 will choose to abate 93 units (from setting $20q_2 = 1860$, which is the firm's cost-minimizing condition, but also from the cost-effective allocation, since the fee of 1860/unit was chosen precisely to achieve cost effectiveness). With the new technology, firm 2 will choose to abate 310 units (again from applying the cost-minimizing condition, but now with the new marginal abatement cost curve). Remember from class that the total cost savings from adopting the technology can be broken into two sources, which are labeled as A and B in the graph. In this case, area A (the savings from lower marginal abatement costs on the first 93 units abated) comes out to $1/2 * (1860 - 558) * 93 = \$60,543$; and area B (the savings from re-minimizing costs by choosing to abate a further 217 units rather than pay fees for emitting them) is $1/2 * (310 - 93) * (1860 - 558) = \$141,267$; and so the total cost savings is $60543 + 141267 = \$201,810$. (It was

also fine if you calculated total abatement costs plus total fee payments with the old technology and then again with the new, and subtracted the latter total from the former.) [8 points]

With the new technology in place, firm 2 will abate 310 units (see above). To know how much will be emitted in the aggregate, we need to find the associated level of emissions for firm 2, and we also need to know how much firm 1 will abate and hence emit. By the familiar formula, emissions equal baseline emissions minus abatement. So firm 2's emissions with the fee and new technology in place will be $344 - 310 = 34$. Firm 1 will try to minimize its costs when faced with the fee, just like firm 2, and again, since the specific fee level was chosen to achieve the cost-effective allocation from part a), it will choose to abate 124 units, since nothing has changed for firm 1. Using the same familiar formula, firm 1's emissions will hence be $321 - 124 = 197$. In the aggregate, then, there will be $197 + 34 = 231$ units emitted with the fee in place and assuming firm 2 has the new technology. The government collects \$1,860 for each of these units emitted, so total fee revenue will be $1860 * 231 = \$429,660$. [4 points]

c) [7 points] Continue to assume that firm 2 has adopted the new technology. Now suppose that the government decides to remove the fee policy, and replace it with a tradable permits system. The permits are initially given away, and divided between the firms such that the original cost-effective abatement allocation found in part a) is obtained. Starting from this allocation, which firm will buy permits, and how many will it buy?

answer:

The initial *permit* allocation, we are told, corresponds with the old (i.e. before firm 2 adopts the new technology) cost-effective *abatement* allocation from part a). So we first need to convert this abatement allocation to emissions. We use the usual equation for that, and in fact, we already did it for firm 1 in the previous part (where it was found that firm 1 emits 197 units). For firm 2, $z_2 = \bar{q}_2 - q_2 = 344 - 93 = 251$. So the initial *permit* allocation is [197, 251]. [2 points]

So, if this is the initial allocation, what is the allocation that we'll eventually end up at? We know from our discussions in class that, whatever the initial allocation, there will be private incentives for the firms to exchange permits until the cost-effective allocation is achieved. We found the cost-effective abatement allocation once already in part a), but this is no longer cost effective, because firm 2 has adopted the new technology. So we first need to apply the equimarginal condition again to find the new cost-effective abatement allocation, and then convert this to emissions. There was no indication that the aggregate abatement target has changed, so we'll continue to treat it as 217. Indeed, this is implicit in the fact that there must have been enough permits printed that the original cost-effective allocation – which of course achieved 217 units of abatement in the aggregate – can be the initial allocation.

$$15q_1 = 6(217 - q_1)$$

$$q_1^{ce} = 62 \rightarrow z_1 = \bar{q}_1 - q_1 = 321 - 62 = 259$$

$$q_2^{ce} = 217 - 62 = 155 \rightarrow z_2 = \bar{q}_2 - q_2 = 344 - 155 = 189.$$

So the emissions (and hence permit) allocation that will eventually be reached is [259, 189]. [4 points]

(Just to check, note that total emissions in each of these allocations are 448, and this corresponds with the aggregate emissions target implied by the relationship between aggregate baseline emissions and the aggregate abatement target: $(321 + 344) - 217 = 448$.)

In the initial allocation, firm 1 holds 197 permits, but for the new cost-effective allocation to be achieved, firm 1 must be holding 259 permits. In other words, the initial allocation forces firm 1 to abate too much compared to what is cost effective. Fortunately, it will be in the interests of firm 1 to *buy* permits until the new cost-effective allocation is reached

(not because it cares about cost effectiveness, of course, but rather because it is in its own cost-minimizing interests to do so by our discussion from class). When all is said and done, firm 1 will buy $259 - 197 = 62$ permits. (And where does it buy these permits from? Firm 2, of course. Firm 2 is holding too many permits in the initial allocation compared to what is cost effective, and it will be in its interests, through bargaining with firm 1, to *sell* $251 - 189 = 62$ permits.) [1 point]

Neither the initial allocation nor the cost-effective allocation that will be achieved through permit trading have anything to do with how firm 2 behaved under the fee. With the fee in place, firm 2 would decide to abate so much that the aggregate abatement target would be exceeded, i.e. there would be too little emitted relative to the government's target. With a permit system, aggregate emissions are fixed by the total number of permits printed – 448 in this case. Thus, it is not feasible with the permit system to replicate the experience with the fee following the technology adoption unless the total number of permits is decreased, which is ruled out in this case by there being enough permits to reach the original cost-effective abatement allocation. Firm 2 still wants to increase its own abatement with the new technology compared to the original cost-effective allocation, but now only to the extent that it can get compensated by selling the permits that this would free up to firm 1; and the price that firm 1 is willing to buy permits for – and hence the incentive to firm 2 to abate more – will be getting lower as we move further down along firm 1's marginal abatement cost curve.

2. [19 points total, 2 parts] Consider a non-renewable natural resource that currently has no substitutes. Society only places value on this resource for the present period and the immediately following period (called periods 1 and 2 respectively), and there will be no exploration for this resource over this time. Marginal benefits to society are represented by the inverse demand function $P_i = 17 - 0.8Q_i$ for each period $i = 1, 2$ (where Q_i is the quantity of the resource that would be extracted/consumed in period i at price P_i dollars per unit) and marginal extraction costs are \$6.00 per unit in each period. The stock of the resource is fixed at 12 units.

a) [12 points] Using a discount rate of 10% where necessary, calculate the dynamically efficient allocation of the resource across the two periods. Verify that the Hotelling Rule holds.

answer:

$$PV(MNB_1) = 17 - 0.8Q_1 - 6 = 11 - 0.8Q_1$$

$$PV(MNB_2) = 1/(1.1) * (11 - 0.8Q_2)$$

For efficiency, $PV(MNB_1) = PV(MNB_2)$

and, of course

$$Q_1 + Q_2 = 12$$

→

$$11 - 0.8Q_1 = 1/(1.1) * [11 - 0.8(12 - Q_1)]$$

$$Q_1^* = 6.369, Q_2^* = 12 - 6.369 = 5.631 \quad [8 \text{ points}]$$

In words, the Hotelling Rule states that the marginal user cost associated with the efficient extraction plan will rise over time at a growth rate equal to the discount rate. This implies that $MUC_2 = (1.1) * MUC_1$, so this is what we need to check.

$$P_1^* = 17 - 0.8(6.369) = 11.9048$$

$$P_2^* = 17 - 0.8(5.631) = 12.4952$$

$$MUC_1 = P_1^* - MC_1 = 11.9048 - 6 = 5.9048$$

$$MUC_2 = P_2^* - MC_2 = 12.4952 - 6 = 6.4952$$

$$MUC_2 / MUC_1 = 6.4952 / 5.9048 = 1.0999865 \approx 1.1, \text{ as required.} \quad [4 \text{ points}]$$

b) [7 points] Suppose, regardless of what you found to be efficient, that 7 of the 12 units available are consumed in period 1. At the beginning of period 2, a recycling technology becomes available. The technology can recover units of the resource that were consumed in the previous period, and make them available for consumption in the second period. However, the technology is costly to use. Specifically, the marginal cost of recycling is given by $MC^R = 6 + 1.2Q^R$, where Q^R is the number of units recycled. From the perspective of period 2 only (because period 1 has already passed), find the efficient consumption of the resource in period 2. Calculate the total recycling costs associated with this efficient consumption level. (There is no need to discount. Marginal extraction costs and marginal benefits are the same as stated previously.)

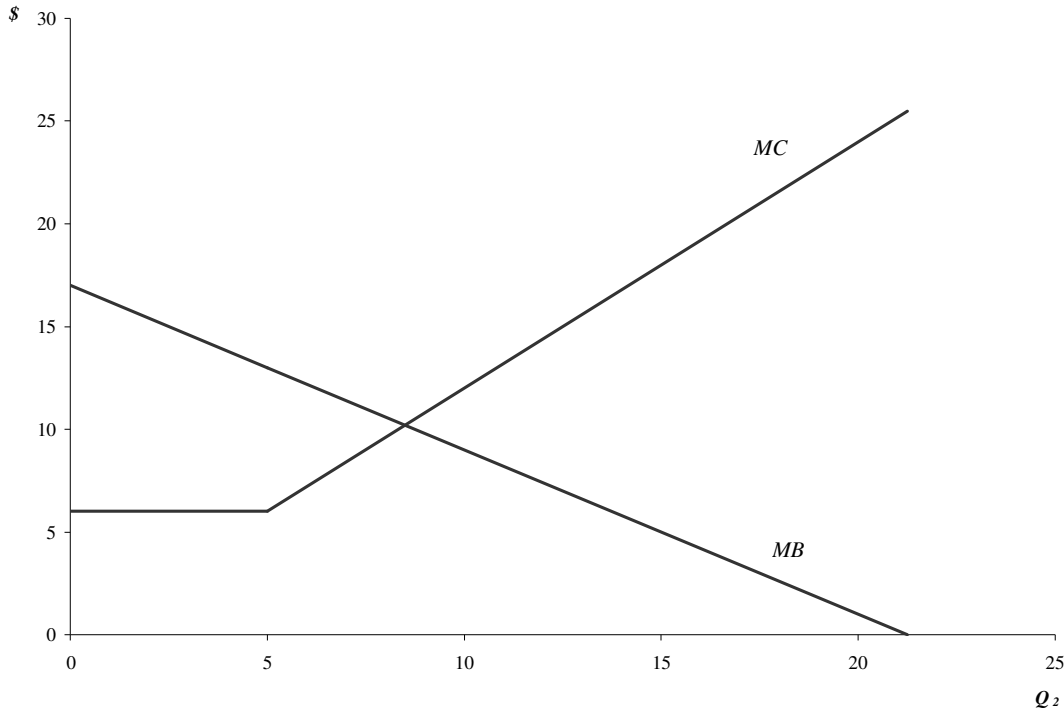
answer:

The way the recycling technology has been described implies that it is able to take some mass of the resource that has been “used”, and convert it into an equivalent mass in good-as-new form. It’s probably easiest to think of the resource as a metal. In the first period, 7 units are extracted and used (to make cars, say). At the beginning of the second period, then, these seven units exist in the form of scrap metal within the junked cars. The recycling technology takes in a unit’s worth of the resource in the form of the scrap metal and outputs a unit of good-as-new metal that can be used for anything a unit of the freshly-extracted resource can be used for. (A more typical assumption is that recycling can never be complete in the sense that some portion of the mass of the used resource is lost in the recycling process – or, equivalently, that the marginal cost of recycling approaches infinity as the proportion being recycled approaches 100%. But that’s more complicated than it needs to be.)

The crucial thing to note is that recycling even one of the units that was consumed in period 1 is more expensive than extracting a fresh unit ($6 + 1.2 \cdot 1 = \$7.2$, which is greater than the constant marginal extraction cost of \$6). So, other things being equal, society would prefer to extract the resource rather than recycle the used units. But extraction only gets us so much, because there are only 5 units left to be extracted (the original total amount available of 12 minus the 7 that it is assumed were used in period 1). Putting this together, society should extract as much of the resource as it wants to, and only then, if it still desires to have more in relation to the cost it will take to get it through the new technology, should it start to recycle. Analytically, this means that efficiency requires $Q^R = Q_2 - 5$ if Q_2 turns out to be greater than 5, or $Q^R = 0$ otherwise.

There are a couple of ways to think about what’s going on. The first one focuses on the cost side. The discussion above implies that the marginal cost of accessing this resource in general is transformed between the fifth and sixth unit, corresponding to the switch in technologies that are used to access the resource (i.e. extraction to recycling). The relationship between Q^R and Q_2 just established can be used to re-write MC^R in terms of Q_2 , thus allowing us to express the costs of extraction and recycling on an equivalent basis: $MC^R = 6 + 1.2Q^R = 6 + 1.2(Q_2 - 5) = 1.2Q_2$ (for $Q_2 > 5$). Now, we can combine this with our marginal extraction cost curve (which expresses the marginal costs of accessing each of the first five units) to form a comprehensive marginal cost curve for

accessing the resource. Refer to the graph below. Marginal benefits are as given previously, and having appropriately formed the marginal cost function, the efficient quantity will be at the intersection of MB and MC .



It's clear that the intersection happens beyond $Q_2 = 5$, so to find it algebraically, we set MB to the appropriate "branch" of MC :

$$17 - 0.8Q_2 = 1.2Q_2$$

$$2Q_2 = 17$$

$$Q_2^{**} = 17/2 = 8.5. \text{ [4 points]}$$

(Another way of thinking about the problem is to treat the market for recycled units separately from the market for new units, with demand in the market for recycled units given by the truncation of aggregate market demand after all of the available extracted units have been sold in the primary market. The results are the same.)

To find total recycling costs at the efficient consumption level, we just need to calculate the area under the marginal cost of recycling curve up to the quantity recycled. But what we have found so far is the efficient level of total consumption of the resource in period 2, including both extracted units and recycled units. To find the number of these units that end up being recycled units, we need to subtract the five extracted units. So $Q^{R**} = 8.5 - 5 = 3.5$. The efficient amount of total recycling costs is thus given by the area under MC^R (in the form depending on Q^R) up to the quantity of 3.5, which will be the area of a rectangle plus the area of a triangle. It should come to $6 \cdot 3.5 + 4.2 \cdot 3.5/2 = \28.35 . (Equivalently, you could use the form of MC^R that depends on Q_2 and calculate the area between the quantities of 5 and 8.5.) [3 points]

BONUS QUESTION [3 points maximum – no penalty for guessing]: Return to the set-up of the first part of the first problem. Firm 2 has the old technology, and there is no fee in place. Suppose the government decides to implement a tradable permits system, with permits initially auctioned. Find the aggregate demand function for permits, where the auction design is such that all firms will face a uniform permit price. Find the permit price that will clear the permit market.

answer:

Let the permit price be p . We know that each firm will minimize costs for a given price p by abating up to the point that $MC = p$ and buying permits at this price for its remaining units emitted. By applying this condition, substituting in the expression for marginal abatement costs and making use of the usual formula to go between units abated and units emitted, we can calculate each firm's demand curve for permits for arbitrary price p . (This is equivalent to our graphical discussion from class.)

$$\begin{array}{l|l} \text{firm 1:} & \text{firm 2:} \\ p = 15q_1 & p = 20q_2 \\ = 15(321 - z_1) & = 20(344 - z_2) \\ = 4815 - 15z_1. & = 6880 - 20z_2. \end{array} \quad [1 \text{ point}]$$

aggregate:

Importantly, permits are private goods, so we need to sum individual demand curves horizontally to get the aggregate demand curve, which entails inverting these functions before summing.

$$z_1 = 321 - (1/15)p, z_2 = 344 - (1/20)p \rightarrow Z = z_1 + z_2 = 665 - (7/60)p. \quad [1 \text{ point}]$$

price:

The market-clearing price is of course where aggregate demand equals aggregate supply. Aggregate supply is fixed at 448 permits (see the calculation in 1c)). So,

$$\begin{aligned} 665 - (7/60)p &= 448 \\ p &= 1860. \end{aligned}$$

This is the same as the fee level found in 1a), which shouldn't be too surprising. A permit system with initial auction will achieve cost effectiveness, as argued in class, and there is only one cost-effective point, namely at the cost-effective allocation where the height of the two firms' marginal abatement cost curves is equal. It was perfectly acceptable to find the price via this argument without having first found the aggregate demand curve for permits. [1 point]