EC371 – Environmental Economics

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Practice Problems for Unit 1: Markets, Efficiency and Market Failure – Solutions

There are three problems. Please read and think about them carefully, and work through them before *looking at the solutions. If you are having trouble, you can seek clarification and help from classmates and during my office hours, but it is highly recommended that you struggle through the questions yourself first. Your goal should be both to learn the mechanics and to grasp the intuition and think more deeply about the issues. Solutions will be posted around the afternoon of Thursday, September 29. If you would like comments on your work and solutions, you can submit them to me at any time.*

1. Consider an economy with two goods, x and y. Denote the price of x as p and set the price of y to 1. There are two consumers, A and B, who have the utility functions $u^A(x^A, y^A) = 10y^A - 0.2(250 - x^A)^2$ and $u^B(x^B, y^B) = y^B - 0.08(62.5 - x^B)^2$ respectively. The aggregate marginal cost curve for producing good x in this economy is given by MC = 4 + 0.016Q where Q is the aggregate quantity of good x. All markets are perfectly competitive, and all of the standard assumptions hold.

a) Find consumer A's demand function for good x (i.e. an expression relating A's consumption of x to p). Find consumer B's demand function for good x as well. (Hint: recall the condition for constrained individual utility maximization from your 201 class or look it up, and note that $MRS^A = 0.04(250 - x^A)$, while $MRS^B = 0.16(62.5 - x^B)$ in this case. You do not have to know how to derive MRS.)

answer:

The condition for constrained utility maximization for an individual is for that individual's MRS to equal the price ratio (or, thinking graphically, the slope of an indifference curve is equal to the slope of the budget frontier, so that the two are tangent to one another). Applying this for A then B (and noting that the ratio of the price of good x to the price for good y is just p in this case, for both consumers):

$0.04(250 - x^A) = p$

$\rightarrow$

$x^A = 250 - 25p$, and

$0.16(62.5 - x^B) = p$

$\rightarrow$

$x^B = 62.5 - 6.25p$.

(You could have alternatively used the utility function and constructed the budget constraint to set up the constrained utility maximization program for each individual and gotten the same results.)

b) Find the aggregate demand function for good x. Verify that the market equilibrium quantity is 125 units and find the associated equilibrium price. Draw a graph showing the aggregate demand and supply curves for good x. Be sure to label your axes, curves, intercepts and equilibrium price and quantity and get the shapes of the curves correct, but don't worry about precise scale.

answer:

$Q = x^A + x^B = (250 – 25p) + (62.5 – 6.25p) = 312.5 – 31.25p$ (aggregate demand)

solve for p: $p = (312.5 – Q)/31.25 = 10 – 0.032Q$ (aggregate inverse demand)

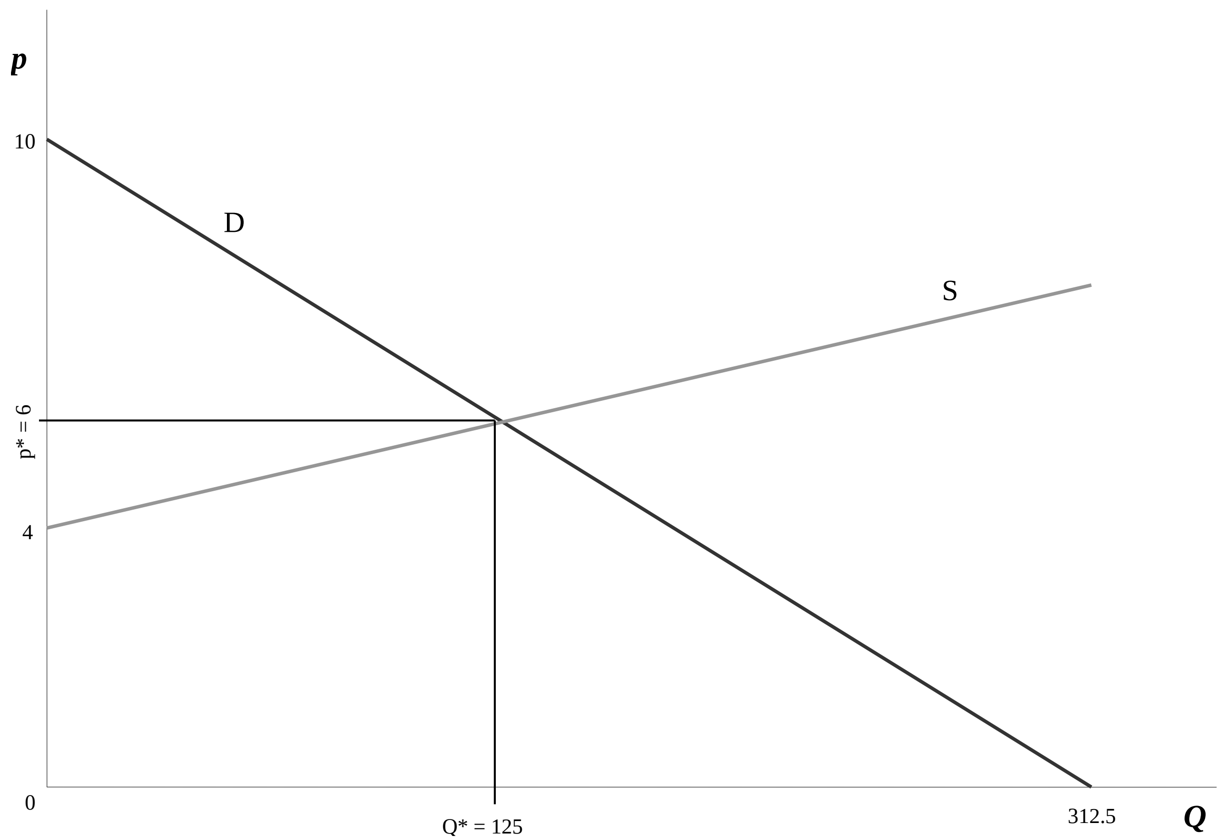
equilibrium: demand = supply (which is given by the aggregate MC)

$10 – 0.032Q = 4 + 0.016Q$

$6 = 0.048Q$

$Q^* = 125$, as required

$p^* = 10 – 0.032Q^* = 10 – 0.032(125) = 10 – 4 = \$6$ (or using the supply function gives 6 as well).



c) Find the economy-wide marginal benefit and marginal cost of the 19th unit consumed/produced. If there were currently 19 units being consumed/produced, would it be good for the economy to decrease consumption/production to 18 units? Explain why or why not.

answer:

I asked this question to get you used to thinking of the demand curve as marginal benefits, to the distinction between marginal and total and to the reasoning behind the desirability of increasing quantity when the marginal benefit of doing so exceeds the marginal cost.

$MB(19) = p(19) = 10 – 0.032(19) = \$9.39$.

$MC(19) = 4 + 0.016(19) = \$4.30$.

Since the marginal benefit of the last unit exceeded the marginal cost, that last unit entailed a net gain for the economy, and so, starting from 18 units, it would make sense to increase consumption to 19 units. Inverting this reasoning, it would not be a good idea to move from 19 units to 18 because this would entail a net loss, i.e. some production cost would be saved, but this would be more than offset by the associated loss in benefits. (Note that to calculate the precise loss from moving from 19 units to 18 you would have to calculate the area between the supply and demand curves between the 18th and 19th units. But for our purposes it is adequate and easier to think in terms of discrete rather than infinitesimal changes for this case.)

d) Calculate total net benefits associated with the equilibrium quantity, the equilibrium quantity plus ten units and the equilibrium quantity minus ten units. (Note that calculating total costs is a bit trickier than it was with the example from class, since the marginal cost curve does not go through the origin in this case. Drawing a graph might help with these calculations.) Based on these calculations, does it appear that the market equilibrium is an efficient outcome? Explain.

answer:

There are a number of mechanical variations on how to calculate net benefits for a given quantity. First, one could calculate total benefits and total costs for each quantity of interest by calculating the areas of the component triangles and rectangles for each quantity. Second, one could calculate producer plus consumer surplus at the equilibrium quantity, then subtract from this the triangles of loss associated with the higher or lower quantity. I am personally most comfortable taking an integral. That's the method I'll use here; if you get the same numbers with whatever method you use, that's great, but if you get different numbers, you should check your method.

Total net benefits corresponding to a certain quantity are calculated as the area between the aggregate demand and aggregate supply curves between a quantity of zero and the quantity of interest (which we can refer to as Z). This is given by the definite integral of the difference between the inverse demand function and the MC curve: $TNB(Z) = \int_0^Z ((10 – 0.032Q) – (4 + 0.016Q))dQ = [6Q – 0.024Q^2]_0^Z$ $= 6Z – 0.024Z^2$. Now, total net benefits for a given quantity can be calculated by substituting the desired quantity into this expression for Z.

$TNB(115) = 6(115) – 0.024(115^2) = 690 – 317.4 = \$372.6$

$TNB(125) = 6(125) – 0.024(125^2) = 750 – 375 = \$375$

$TNB(135) = 6(135) – 0.024(135^2) = 810 – 437.4 = \$372.6$

It looks like the equilibrium quantity of 125 yields the highest possible level of total net benefits, since increasing or decreasing the quantity both lead to lower levels of TNB. Therefore, it appears that the market equilibrium is efficient, as these calculations tell us (at least heuristically) that total net benefits are maximized – which is required for efficiency – at the equilibrium quantity. This is what we would expect given that we assumed that "all of the standard assumptions hold" at the beginning of the question:

there are no conditions (like externalities) that would lead to market failure, so the market outcome should be efficient.

Additional thoughts on Problem 1.

In addition to efficiency, we can also use the model to think a bit about equity. One way to do this is to compare how well off each consumer is in equilibrium using utility levels. For example, assume that each consumer has an income of $1,200. Find the quantities of goods *x* and *y* that are consumed by each individual at the equilibrium price, making use of their individual demand curves for good *x* from part a) and their budget constraints. Then plug these quantities into the utility functions.

$x^A(6) = 250 - 25(6) = 100$, and $x^B(6) = 62.5 - 6.25(6) = 25$. (You can check that these add up to the aggregate equilibrium quantity, which they do in this case: $100 + 25 = 125 = Q^*$.)

The general form of the budget constraint for each consumer in this case is $px + y = 1200$, so rearranging and focusing on each consumer individually, $y^A(6) = 1200 - 6x^A(6) = 1200 - 600 = 600$ and $y^B(6) = 1200 - 6x^B(6) = 1200 - 150 = 1050$.

$u^A(100\ ,\ 600) = 10(600) - 0.2(250 - 100)^2 = 6000 - 4500 = 1500$

and

$u^B(25\ ,\ 1050) = 1050 - 0.08(62.5 - 25)^2 = 1050 - 112.5 = 937.5$.

In this case, A's utility level is clearly greater than that of B. So we can conclude that, on a utility basis, the equilibrium entails some inequality. Whether or not in*equality* in utility is considered in*equitable* (i.e. unfair) ultimately depends upon society's sentiments about justice etc.

The point is that efficiency (which we'll be talking a lot about this semester) may not be the only thing or even the most important thing that societies and governments care about, and that achieving efficiency is not always sufficient for achieving these other goals.

Of course, efficiency is not necessarily always at odds with other goals either: if the society in this question had a large enough tolerance for inequality, it might very well be the case that this market outcome would be judged as being fair, in addition to meeting our definition of efficiency.

How do we know what society judges to be equitable in real life? In short, we don't. In social choice theory we typically work with a hypothetical construct called a social welfare function, which aggregates individual utility functions in a way that embeds postulated preferences for equity. However, it's not necessary to get into this for our purposes in this class, as our occasional discussions of fairness and other objectives won't require such precision.

I should note that there is some disagreement as to whether it can ever be proper in a theoretical sense to compare utility levels across people. I can provide references on this subject to interested readers, but for the sake of making the general point about fairness and efficiency, it is safe to take it for granted that the specific utility functions have been appropriately chosen so as to make comparison possible. One could alternatively have used individual consumer surpluses rather than utility levels to make a similar welfare comparison.

If we did know society's preferences for equity, we might be able to construct an outcome that is fair according to these preferences through redistribution. For simplicity, let's say that society requires both individuals to be equally "well off" in a utility sense and would consider any other outcome to be unfair. If we had this information, we could say that the market equilibrium is unfair because it involves some utility inequality. But if the government could take about $51.14 from A and give it to B without destroying any of the total income or output in the economy, the equilibrium that would follow – which you can see will still involve an aggregate quantity of 125 and still be efficient if you try it (as, actually, nothing changes in parts a) through d)) – will be judged as fair because now each individual will have the same level of utility. This, in a nutshell, is a demonstration of what's called the Second Theorem of Welfare Economics, to go along with the First, which was demonstrated in part d). The problem is that taxation often dulls incentives to work and invest, so that taxation is not generally costless, potentially exerting a trade-off between achievement of efficiency and achievement of equity (which takes a more complicated model and a general equilibrium sense of efficiency to demonstrate fully).

The utility functions used in this problem were chosen because of the simplicity of the resulting demand functions, but are not of a form that economists typically use. For a challenge, you might want to follow the parts of this question with the following changes: each consumer has the utility function $u(x, y) = 2\ln(x) + \ln(y)$ (where ln represents the natural logarithm function); consumer A has an income of $51 and B has an income of $30; the aggregate marginal cost curve for producing good x in this economy is given by $MC = 2Q^2$; and the prices are as above. In this case, to get the individual demand curves for x, you will need to use both the MRS condition and the budget constraint for each consumer, and you will see that the demand curves are not straight lines (nor is the given MC curve). You should get an equilibrium quantity of 3. You must use integration to calculate total net benefits, and in addition you will have to use a lower quantity limit of 1 rather than 0 so that you do not get infinite results. This is just being suggested for those who might be planning to go further in economics or are otherwise interested, and is not required.

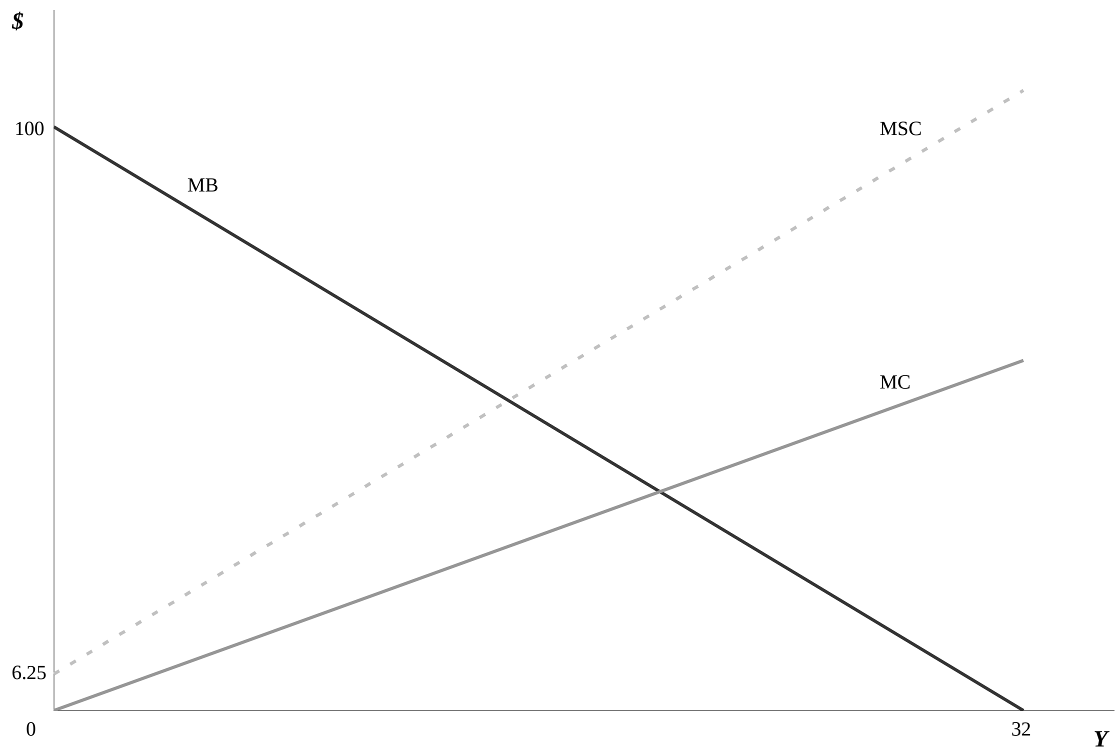
2. Suppose that a wood pulp mill is situated on a bank of the Charles River. The private marginal cost (MC) of producing wood pulp (in $ per ton) is given by $MC = 1.875Y$ where Y is tons of wood pulp produced. In addition to the private marginal cost, an external marginal damage (MD) is incurred due to harm from pollutant flows into the river, valued at $6.25 + 1.25Y$, in $ per ton. This external cost is borne by the wider community, not by the polluting firm in isolation. Even though we are concerned with a single mill, it behaves perfectly competitively. The aggregate inverse demand curve for wood pulp, in $ per ton, representing both the private and social marginal benefits (MB), is given by $MB = 100 – 3.125Y$.

a) Calculate the marginal social cost (MSC) curve and draw a diagram illustrating the MC, MB and MSC curves. (Be fairly precise with the graph: label your axes, intercepts and curves; don't worry too much about getting the scale perfect, but do make sure the shapes, positions and slopes of the curves are roughly correct in relation to one another.)

answer:

$MSC = MC + MD = 1.875Y + (6.25 + 1.25Y) = 6.25 + 3.125Y$.

The graph (next page) should make clear that the MSC curve both has a positive vertical intercept and is steeper than the MC curve. The MC curve should be noticeably less steep than the MB curve (in absolute value).



b) Find the market equilibrium quantity of wood pulp in tons, i.e. the quantity that maximizes total private net benefits. Find the socially efficient pulp output, i.e. the level that maximizes total social net benefits. Explain why you would expect one of these output levels to be larger than the other. (You should get nice round numbers for these quantities, and the difference between them should be five units.) Calculate the efficiency loss that would be suffered if the market rather than the efficient quantity were produced.

answer:

market: MC = MB
$1.875Y = 100 - 3.125Y$
$5Y = 100$
$Y^* = 20$

efficient: MSC = MB
$6.25 + 3.125Y = 100 - 3.125Y$
$6.25Y = 93.75$
$Y^{**} = 15$

We would expect the efficient output level to be lower than the market output level (which is what we have found). This is because, as usual with a negative externality, private producers and consumers do not account for broader social costs when making their decisions. Since private costs are lower than social costs, private actors will demand and supply too much relative to the efficient outcome.

Total net social benefits (the area between MB and MSC up to a given quantity) are maximized at the intersection of MB and MSC (by definition), and at the market output of 20, the additional triangle between MSC and MB from a quantity of 15 to 20 is an efficiency loss. The area of this triangle – and hence the dollar value of the loss – can be found geometrically or by integrating. I'll again use calculus because it's what I'm comfortable with, but you can use whatever method you'd like as long as it gives you the right answer.

$$\text{loss} = \int_{15}^{20} [6.25 + 3.125Y - (100 - 3.125Y)]\mathrm{d}Y$$
$$= [3.125Y^2 - 93.75Y]_{15}^{20}$$
$$= 3.125\times400 - 93.75\times20 - 3.125\times225 + 93.75\times15$$
$$= \$78.125.$$

It is probably easiest to visualize this on the graph. It never hurts to practice drawing it again. The "height" of the triangle that we are interested in is the horizontal distance (20 – 15) and the "base" of the triangle is the vertical distance (MSC(20) – MB(20)).

c) Calculate the Pigouvian per-unit tax to be collected from the mill that would achieve the socially efficient output level. Calculate the tax revenue that would be generated by implementing this tax.

answer:

t^{**} = MSC(Y^{**}) – MC (Y^{**}) (or, equivalently, MB – MC at Y^{**} or MD at Y^{**})
$= 6.25 + 3.125\times15 - 1.875\times15$
$= \$25$

revenue = $t^{**}\times Y^{**} = 25\times15 = \$375$.

d) Suppose that, instead of putting the tax policy in place, the Massachusetts government determines that the wood pulp mill has the right to pollute the river as much as it wants to, and commits to legally enforcing that right. Is it theoretically possible to achieve social efficiency in this situation without any further government involvement? Do you think efficiency would likely be achieved in reality?

answer:

The Coase Theorem tells us that the socially efficient output level can be achieved in this circumstance whether the mill or other stakeholders (e.g. universities located along the river) are granted property rights (under certain other conditions, which we'll get back to in a moment). For each unit of output between 15 and 20, there should be incentives for both parties (the affected community on one hand and the firm on the other) to bargain and arrive at the socially efficient level of output. In reality, there are likely to be too many stakeholders involved and legal fees required to be able to organize the bargaining process successfully, probably leading to no bargaining and the status quo outcome of $Y = 20$. In the worst case, other polluters may even be attracted to the area, or the original firm encouraged to expand by the awarding of property rights.

Additional thoughts on Problem 2.

Even though, with the Pigouvian tax from part c) in place, it is the firm that bears what we call the "administrative burden" of the tax, consumers are affected too, because they pay a higher price for paper than they would with no tax in place. We can calculate what is called the "effective burden" of the tax that falls on the firm (defined as the difference between the price-per-unit received by the mill without the tax in place and that effectively received with the tax in place, as a proportion of the tax level) and on consumers (defined as the difference between the price-per-unit paid by consumers with the tax in place and that without the tax in place, as a proportion of the tax level).

prod. burden: $[\text{MC}(Y^*) - \text{MC}(Y^{**})]/t = [1.875\times20 - 1.875\times15]/25 = 9.375/25 = 37.5\%$.

cons. burden: $[\text{MB}(Y^{**}) - \text{MB}(Y^*)]/t = [(100 - 3.125\times15) - (100 - 3.125\times20)]/25 = 15.625/25 = 62.5\%$.

The producers bear the administrative burden, but the "internalization of the externality" actually ends up being shared by society as a whole – in fact, in this particular case, with consumers bearing relatively more of the overall tax burden. In a sense, this is appropriate, since it is the behavior of firms and consumers *together* that determines the market output level of the polluting good.

(Be sure to draw a graph and/or consult a micro textbook to see why these prices make sense. In essence, consumers pay a higher price inclusive of tax with the tax in place than when it wasn't in place, even though it is the firm that is administratively responsible for remitting the tax. Once the firm remits the tax receipts, it effectively receives a lower price than it would have before the tax was levied. The price without the tax in place for both producers and consumers is of course the price at the market equilibrium, i.e. the vertical height at which MB and MC intersect.)

It's also possible to place the administrative burden of submitting the tax on consumers rather than producers. The same tax level found in part c) also achieves social efficiency when it is collected from consumers (which causes the demand curve to shift down – refresh your micro if you're having trouble understanding why). However, a result that you should have seen somewhere in your previous economics classes is that, regardless of which side bears the administrative burden, the output level and the effective burdens will not change.

market equilibrium with tax on consumers: $\text{MC} = \text{MB}_{\text{with tax}}$
$1.875Y = 100 - 3.125Y - t$
$1.875Y = 75 - 3.125Y$ (t=25 from before)
$5Y = 75$
$Y^{***} = 15 = Y^{**}$ as required.

(This is mathematically equivalent to setting $\text{MC}_{\text{with tax}} = \text{MB}$, so the result is unsurprising. Convince yourself that the effective burden on consumers is still 62.5%.)

Now suppose that, instead of a tax of any kind, the government institutes a subsidy on every unit *not* produced up to and including, say, the 28th unit. This policy will also achieve social efficiency when the subsidy level is the same as the tax level found in part c).

A per-unit subsidy for *not* producing has the effect of shifting the MC curve *up*. The intuition for this is that the subsidy injects an opportunity cost for producing additional units, since for each additional unit produced, the firm has to forego a subsidy payment. To think about this mechanically, you could form the firm's total cost curve with and without the subsidy in place, then take a derivative to get the corresponding marginal cost curve. *Total* costs are reduced by the amount $s(25 - Y)$ with the subsidy in place, so as the firm's choice of output Y rises and gets closer and closer to 25, the amount by which total costs are reduced is getting smaller. Or in other words, *marginal* costs are *increased* by the amount s. (To be precise, the $\text{MC}_{\text{with subsidy}}$ curve will jump down and rejoin the MC curve beyond a quantity of 28, but this is outside the range we need to worry about.)

market equilibrium with subsidy to firms for not producing: $\text{MC}_{\text{with subsidy}} = \text{MB}$
$1.875Y + s = 100 - 3.125Y$

$1.875Y + 25 = 100 – 3.125Y$
$5Y = 75$
$Y^{****} = 15 = Y^{**}$ as required.

(Again, the equilibrium condition with the subsidy in place is mathematically equivalent to that with a tax on consumers in place, as well as to that with a tax on producers in place.)

A potential problem with this option is that it involves the government giving money away, so that it would have to first raise that revenue through other means. To figure out how much the government will pay in subsidies in total in a situation like this, we need to find out how many units the firm ends up *not* producing below the limit. If the firm *does* produce 15 units, it *doesn't* produce 28 – 15 = 13 units.

subsidy payments = $s \times 13$ = \$325.

The point is that there are a bunch of variations on the baseline Pigouvian taxation scenario, but they all leave the essentials (achievement of the socially efficient output level, and the internalization being shared between consumers and producers) untouched. Choosing which variation to pass as law (or not) usually boils down to a lot of politics and, I might suggest, playing on voters' misunderstandings and desires to punish who they see as the "bad guys". For a bit more on such misunderstandings, here is an example from an old post on Greg Mankiw's blog: http://gregmankiw.blogspot.com/2008/01/taxes-dont-stay-where-you-put-them.html.

A final point is that, if we had some information (perhaps utility functions) for the individuals underlying the aggregate demand curve in this example, we could think more about the equity implications of addressing the externality through taxation. For example, if people with lower incomes tend to have preferences that lead them to consume more paper than others on average, imposing the tax would probably lead to greater utility inequality between those with high incomes and those with low incomes than would exist without the tax, which probably wouldn't make the tax very desirable from a fairness perspective.

3. A group of northeastern states is conducting research on auction designs for a regional carbon dioxide cap-and-trade system. The marginal costs for conducting such research are flat, at \$31 per month in thousands of dollars. The marginal benefits enjoyed by the region are derived from the grants they receive as well as the usefulness of the results, and depend on how much research is being done: MB = 50 – 0.5Q, where Q is months of research done in the group of northeastern states and MB is measured in thousands of dollars. There are three other states or groups of states that are also considering implementing a cap-and-trade system: California, a group of southern states and a group of western states. These states enjoy some benefit from the research done by the northeastern states, since those research findings can partially inform the auction design process for any region. If they could buy months of research done by the northeastern states for some price p (in thousands of dollars per month), their demand curves would be $q^c = 100 – 20p$, $q^s = 100 – 10p$ and $q^w = 100 – 8p$ for California, the southern states and the western states respectively, where q^i refers to the number of months of research done by the northeastern states demanded by state/region i.

a) Think of the research done by the northeastern states as a public good. Find the aggregate marginal willingness to pay relation for this research amongst the three state/regions aside from the northeastern region.

answer:

Since research is a public good, to get aggregate marginal willingness to pay we need to sum the individual marginal willingnesses to pay across the relevant parties. That means we first need to invert the individual demand curves, then do the summation (i.e. sum prices/marginal willingnesses to pay for a given quantity that will be consumed simultaneously). This is what we called vertical summation in class.

$q^c = 100 - 20p \rightarrow MWTP^c = 5 - 0.05Q$
$q^s = 100 - 10p \rightarrow MWTP^s = 10 - 0.1Q$
$q^w = 100 - 8p \rightarrow MWTP^w = 12.5 - 0.125Q$

$MWTP = MWTP^c + MWTP^s + MWTP^w = 27.5 - 0.275Q$.

(It is appropriate to substitute the individual q's for Q because the research done by the northeastern region can be "consumed" by all four regions simultaneously.)

b) Now think of the research done by the northeastern region in terms of a positive externality. Call the marginal willingness to pay relation derived in part a) the marginal external benefit (MEB), and use this to find a marginal social benefit (MSB) relation encompassing the marginal internal benefits and the marginal external benefits arising from research done by the northeastern region. Calculate the amount of research that will be done if the northeastern region acts in its self interest. Calculate the amount of research that is efficient from the perspective of all four state/regions. Give some intuition for why the efficient outcome is higher than the self-interest outcome. (You should get 60 for the socially efficient level. Be careful when converting the verbal description of MC into mathematical form.)

answer:

$MEB = MWTP_{\text{other regions}} = 27.5 - 0.275Q$

$MSB = MB + MEB = (50 - 0.5Q) + (27.5 - 0.275Q) = 77.5 - 0.775Q$.

self-interest: MB = MC
$50 - 0.5Q = 31$
$0.5Q = 19$
$Q^* = 38$.

(Note that MC is NOT $31Q$. Think carefully about the description of marginal costs, and you should be able to reason out why this wouldn't make sense. You won't lose many points if you make this kind of a mistake on an exam and carry it through correctly for the rest of the problem, but it's worth taking note of now.)

socially efficient: MSB = MC
$77.5 - 0.775Q = 31$
$0.775Q = 46.5$
$Q^{**} = 60$.

The socially efficient level is higher than the self-interest level because there is a positive externality at play here. The northeastern region only considers its private benefits derived from the research, not the larger social benefits. (It seeks to maximize the difference between its total private benefits and its total private costs, which occurs at MB = MC.) No individual (region, person, etc.) would be willing to bear an additional cost for units for which its private benefits were not large enough.

c) Suppose that the federal government decides to offer a subsidy to the northeastern region to try to increase the amount of research done. Find the appropriate per-unit Pigouvian subsidy that will achieve the efficient amount of research. Calculate the total subsidy amount that the federal government will disburse to the northeastern region with this policy in place. Calculate the associated efficiency gain.

answer:

You can either think of the subsidy as shifting up the MB curve or shifting down the MC curve. I find it a little more natural to think in terms of the MC curve, but either way is fine. (Note that this is the usual kind of subsidy, paid on units produced, hence shifting marginal costs down. This is in contrast to the strange subsidy for *not* producing from the additional thoughts to the previous problem, which shifted marginal costs up, as discussed there.) The key is that we want the subsidy to be large enough to erase the difference between MB and MSB *at the efficient output level*. What is this difference?

$s^{**} = \text{MSB}(Q^{**}) - \text{MB}(Q^{**}) = [77.5 - 0.775\times60] - [50 - 0.5\times60] = 31 - 20 = \$11$ (in thousands).

(Note that MSB and MC are equal at Q^{**}, by definition, so you can calculate the Pigouvian subsidy equivalently as $\text{MC}(Q^{**}) - \text{MB}(Q^{**})$. And, analogous to the negative externality case where you can use MD, you can also calculate the Pigouvian subsidy equivalently as $\text{MEB}(Q^{**})$.)

total subsidy payments = $s^{**}\times Q^{**} = 11\times60 = \$660{,}000$ (since everything is measured in thousands of dollars in this question, and the per-unit subsidy is paid on each unit produced).

I was being a bit tricky by asking you about the efficiency gain and the total subsidy payment in the same part. They don't actually have anything to do with one another, since the subsidy payment is just some pure transfer from the government (who presumably got the money through taxation of some sort that is happening outside of what concerns us for the situation at hand) to the researchers. Having the per-unit subsidy in place is crucial, because it aligns incentives such that the efficient outcome will be achieved through subsequent private actions. But the total amount of subsidy payments that results does not in itself affect efficiency.

The level of social efficiency is just, as usual, the area between the appropriate marginal benefit curve and the appropriate marginal cost curve up to a given quantity. In this case, the appropriate curves are MSB and MC. The quantities of concern are Q^* (the quantity achieved without the subsidy in place) and Q^{**} (the socially efficient quantity, achieved with the subsidy).

The efficiency improvement will hence be given by the triangle between the MSB and MC curves from a quantity of 38 to a quantity of 60. I used calculus to calculate an area last time, so I'll just use the geometric definition this time.

gain = $\text{h}\times\text{b}/2 = [\text{MSB}(Q^*) - \text{MC}(Q^*)]\times[Q^{**} - Q^*]/2 = [77.5 - 0.775\times38 - 31]\times[60 - 38]/2 = 187.55 =$ \$187,550 (because MSB and MC are measured in thousands of dollars).

(Note that, even though I didn't ask you to draw a graph in this question, doing so might help to visualize what's going on and otherwise be good practice. Note also that there are possible variations on the subsidization policy, akin to the additional points made regarding the previous problem.)

This question is fictional for the most part, but it was loosely inspired by reality: http://www.enn.com/ecosystems/article/38266. We'll be talking much more about tradable permits and "cap-and-trade" systems during our air pollution unit.