

## EC371 – Environmental Economics

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### Practice Problems for Unit 2: Cost-Benefit Analysis – Solutions

*There are three problems. Please read and think about them carefully, and work through them before looking at the solutions. If you are having trouble, you can seek clarification and help from classmates and during my office hours, but it is highly recommended that you struggle through the questions yourself first. Your goal should be both to learn the mechanics and to grasp the intuition and think more deeply about the issues. Solutions will be posted around the afternoon of Thursday, October 13, 2011. If you would like comments on your work and solutions, you can submit them to me at any time.*

1. Consider a coastal area that is currently not in use and on which society does not place intrinsic value. The government is considering developing the area, and has two projects in mind. The first project is to build a marine amusement park. The initial construction cost of the park will be \$970, and there will additionally be annual maintenance costs of \$130 each year that the park operates after the year in which it is built. The government also wants to consider the fact that people who use the park will not undertake other forms of leisure that they would have undertaken in the absence of the park; they estimate the value of this foregone recreation as \$20 per year beginning the year after the park is built; this represents an additional recurring annual cost. The government expects that \$300 in tourism revenue will be earned per year beginning the year after the park is built, representing the recurring annual benefit of the project. The second project is to mine the coral reef off of the coast for limestone. The extraction cost (paid once in year 0) is \$3,200, and revenues of \$4,500 are realized once only, immediately in year 0. However, the removal of the reef will set in motion a process of continuous coastal erosion, and this damage is estimated at \$200 per year beginning the year after extraction. (The two projects are mutually exclusive because the removal of the reef will destroy the value of the area for marine recreation.)

a) Suppose the government sets a 10-year time horizon (i.e. years 1 through 10 inclusive, but also considering the initial year 0) for comparing the two projects. (The government places zero weight on future periods in which a new administration will be in place.) Suppose further that the government requires use of a discount rate equal to 5% for performing cost-benefit analyses. Calculate the net present value of the two projects. On this basis, would you recommend that the government undertake one project over the other? Is either of the projects preferred to the status quo according to the net present value criterion?

answer:

$$\delta = 1/(1 + r) = 1/(1.05) = 0.9524.$$

$$\begin{aligned} \text{NPV}_{\text{park}} &= -C_0 + \frac{\delta - \delta^{11}}{1 - \delta} \text{NB} \\ &= -970 + \frac{0.9524 - 0.9524^{11}}{1 - 0.9524} (300 - 130 - 20) \\ &= -970 + 7.7225(150) \\ &= \$188.38. \end{aligned}$$

$$\begin{aligned}
NPV_{\text{mine}} &= -C_0 + B_0 + \frac{\delta - \delta^{11}}{1 - \delta} NB \\
&= -3200 + 4500 + \frac{0.9524 - 0.9524^{11}}{1 - 0.9524} (-200) \\
&= 1300 + 7.7225(-200) \\
&= -\$244.50.
\end{aligned}$$

On this basis, we would recommend building the park rather than undertaking the mining project. The park yields positive net present value, and the status quo is zero (because society places no intrinsic value on the coastal area and it is not currently in use), so the park is also preferred to the status quo.

b) Suppose instead that the government requires use of a 10% discount rate. Still using the 10-year time horizon, re-calculate the net present value of each project. Would you make a different recommendation to the government compared to part a) based on this calculation? Give some intuition about why this is the case.

answer:

$$\delta = 1/(1 + r) = 1/(1.1) = 0.9091.$$

$$\begin{aligned}
NPV_{\text{park}} &= -C_0 + \frac{\delta - \delta^{11}}{1 - \delta} NB \\
&= -970 + \frac{0.9091 - 0.9091^{11}}{1 - 0.9091} (300 - 130 - 20) \\
&= -970 + 6.1449(150) \\
&= -\$48.27.
\end{aligned}$$

$$\begin{aligned}
NPV_{\text{mine}} &= -C_0 + B_0 + \frac{\delta - \delta^{11}}{1 - \delta} NB \\
&= -3200 + 4500 + \frac{0.9091 - 0.9091^{11}}{1 - 0.9091} (-200) \\
&= 1300 + 6.1449(-200) \\
&= \$71.02.
\end{aligned}$$

The recommendation is reversed from the situation in part a): now the mining project is preferred to the status quo and to the park. The intuition for this result is that the mining project has large up-front benefits and recurring costs into the future, while the park has a large up-front cost and recurring (net) benefits into the future. With a higher discount rate we place less value on the future, so with a 10% discount rate we place less weight on the recurring costs of the mining project and also on the recurring

benefits of the park than was the case with a 5% discount rate. The higher discount rate leads us to recommend projects for which benefits are realized closer to the present.

c) Now suppose that the government cares about all future periods and so requires the use of an indefinite time horizon. Calculate the net present value of each project using both a 5% and a 10% discount rate. Does the choice of these two discount rates have any effect on the recommendation that you would give to the government on the basis of the net present value criterion?

answer:

$$\begin{aligned} \text{NPV}_{\text{park},5\%} &= -C_0 + (1/r)\text{NB} \\ &= -970 + (1/0.05)(300 - 130 - 20) \\ &= -970 + 20(150) \\ &= \$2030. \end{aligned}$$

$$\begin{aligned} \text{NPV}_{\text{mine},5\%} &= -C_0 + B_0 + (1/r)\text{NB} \\ &= -3200 + 4500 + (1/0.05)(-200) \\ &= 1300 + 20(-200) \\ &= -\$2700. \end{aligned}$$

$$\begin{aligned} \text{NPV}_{\text{park},10\%} &= -C_0 + (1/r)\text{NB} \\ &= -970 + (1/0.1)(300 - 130 - 20) \\ &= -970 + 10(150) \\ &= \$530. \end{aligned}$$

$$\begin{aligned} \text{NPV}_{\text{mine},10\%} &= -C_0 + B_0 + (1/r)\text{NB} \\ &= -3200 + 4500 + (1/0.1)(-200) \\ &= 1300 + 10(-200) \\ &= -\$700. \end{aligned}$$

We would recommend the park over both the status quo and the mining project regardless of the discount rate used. Since the government now cares that the costs of coastal erosion from the mining project will be accrued indefinitely, it would take a very high discount rate (very little weight put on the future) to justify the mining project.

2. (Adapted from *Economics and the Environment* by Eban S. Goodstein.) Consider a hotel owner who is taking a 40-year perspective on renovating the hotel. There are two options for light bulbs to be used in each of the 1,000 fixtures in the hotel. Suppose that each fluorescent bulb costs \$15 and each incandescent bulb costs \$1, and that fluorescent bulbs last 40 years but incandescent bulbs last only 4

years. The annual electricity cost of using 1,000 incandescent bulbs would be \$2,800 while the annual electricity cost of using 1,000 fluorescent bulbs would be \$800.

a) What (conceptually) would the hotel owner want to use for a discount rate and why? Supposing that the owner decides to use a discount rate of 5%, calculate the present value of costs for each type of light bulb. The annual electricity cost will be borne each year starting in year 1 and continuing through year 40. Bulbs will be purchased once, in year zero (either the one set of fluorescents or the ten sets of incandescents required for the full time horizon). On the basis of a lower present value of costs, what type of bulb would you recommend that the hotel owner install?

answer:

Since the hotel is a private business, the relevant market interest rate (in this case, the return on a 40-year secure bond, for example) should be used as the discount rate. This is the appropriate outside option or opportunity cost of funds. (The bank account logic we talked about in class doesn't go through quite as nicely because the problem has been simplified to consider costs only and not the stream of revenue that the owner is presumably receiving while the hotel is in operation. But what I was looking for here was just a statement of our general rule: when it's a private, individual decision, the appropriate discount rate is a market interest rate.)

Note: everything is a cost in this problem, so everything should have a negative sign in front. But it's fine to ignore this as long as we do it uniformly in a problem with only costs. For neatness I've dropped all the minus signs; the important thing is that we want to pick the type of bulb that yields a *lower* present value of costs in absolute value.

$$\begin{aligned} PV_{fl} &= C_{0,fl} + \frac{\delta - \delta^{41}}{1 - \delta} C_{fl} \\ &= 15 * 1000 + \frac{0.9524 - 0.9524^{41}}{1 - 0.9524} (800) \\ &= 15000 + 17.1591(800) \\ &= \$28,727. \end{aligned}$$

$$\begin{aligned} PV_{inc} &= C_{0,inc} + \frac{\delta - \delta^{41}}{1 - \delta} C_{inc} \\ &= 10 * 1 * 1000 + \frac{0.9524 - 0.9524^{41}}{1 - 0.9524} (2800) \\ &= 10000 + 17.1591(2800) \\ &= \$58,045. \end{aligned}$$

We would prefer the fluorescent bulbs, since the present value of costs is lower.

(As an aside, the problem could have been structured slightly differently by specifying that the owner, if choosing incandescents, would buy the 1000 bulbs once every four years as required, rather than all ten sets up front. If this had been the case, we would not have been able to use the formulas we've developed

so far, or at least not very straightforwardly. But we could still make the calculation, just with a bit more work. To illustrate, assume the owner would buy the bulbs in years 0, 4, 8, and so on up to 36, using the newly-purchased bulbs for the subsequent four years.

$$\begin{aligned}
 PV_{\text{inc}} &= PV_{\text{inc,bulbs}} + PV_{\text{inc,elec.}} \\
 &= (1 + \delta^4 + \delta^8 + \delta^{12} + \dots + \delta^{36})C_{\text{bulbs,every4years}} + \frac{\delta - \delta^{41}}{1 - \delta}C_{\text{elec}} \\
 &= \frac{1 - 0.9524^{40}}{1 - 0.9524}(1 * 1000) + \frac{0.9524 - 0.9524^{41}}{1 - 0.9524}(2800) \\
 &= 4.8391(1000) + 17.1591(2800) \\
 &= \$52,885.
 \end{aligned}$$

To calculate the sum  $S = (1 + \delta^4 + \delta^8 + \delta^{12} + \dots + \delta^{36})$  I used a slight modification of the algebraic summation trick I showed you in class – specifically, calculating  $\delta^4 S$ , subtracting from  $S$  on both sides of the equations and solving for  $S$  – to get the short expression for the sum given above. You could also just have calculated the sum of the ten terms directly by brute force. There is also a way to transform the problem by making a “change of variables” that would allow us to get the same result with more familiar formulas. Finally, if we were in a hurry, we could approximate by allocating the bulb cost borne every four years equally across those four years, which would allow us to use our same old formula; but this would not give us the exact answer.

In any case, none of what has been presented in this aside is required for the course, but I do think it can be good practice to visualize the structure of a given problem from “first principles” now and then rather than always using our pre-packaged formulas by force of habit alone. In reality, projects we might be called on to evaluate are rarely as cleanly structured as the examples we work with in this class, but we’d be using one of several specially-designed software packages available rather than these formulas to complete our analysis anyway. To eventually be able to use such software packages effectively, though, requires an understanding of the fundamentals of what they calculate and how, and it is this kind of understanding that these examples are meant to help you obtain.

We would prefer the fluorescent bulbs under this slightly-modified structure of the problem too (because nothing has changed for fluorescents, and  $53k > 29k$ ). However, the present value of costs for incandescents is slightly lower under this structure than under the initial structure we considered. This makes sense, since this structure has purchases delayed until various future periods – which we value less than purchases made in the present – rather than specifying them as all being up front, as in the initial structure.)

b) Repeat the calculation, but this time consider the possibility that there is some uncertainty in the energy efficiency of incandescent bulbs. Specifically, suppose that there is a 60% chance that the bulbs will be of regular efficiency (annual electricity costs will be \$2,800) and a 40% chance that the bulbs will be of a higher efficiency (annual electricity costs will be \$1,300). Does this change your recommendation?

answer:

$$EV(\text{annual electricity cost}) = 0.6 * 2800 + 0.4 * 1300 = \$2200$$

$$\begin{aligned}
PV_{\text{inc}} &= C_0 + \frac{\delta - \delta^{41}}{1 - \delta} (\text{EV}(\text{annual electricity cost})) \\
&= 10 * 1 * 1000 + \frac{0.9524 - 0.9524^{41}}{1 - 0.9524} (2200) \\
&= 10000 + 17.1591(2200) \\
&= \$47,750.
\end{aligned}$$

The fluorescent light bulbs are still preferred (29k < 48k). This is a simple but approximate way to deal with the possibility that there will be some sort of technical improvement in incandescent bulbs over the 40-year horizon. (Of course, there are lots of other things that are uncertain that we have not dealt with, such as the possibility that incandescent bulbs will stop being available at some point in the future or that an additional option like LEDs becomes available, as it appears may actually happen in the near future: <http://search.ebscohost.com.ezproxy.bu.edu/login.aspx?direct=true&db=rlh&AN=37042545&site=ehost-live>. There are techniques that can be used to attempt to address concerns like these, but ultimately we have to accept that there will be some imprecision/incompleteness in our analysis, especially with projects of a very long time frame, associated with not being able to predict the future perfectly.)

c) Now suppose that, instead of a private hotel owner, an identical decision is being made by the government concerning, for example, a building that is open to the public. Will the government want to do anything differently in undertaking the cost analysis?

answer:

We haven't talked about the kinds of things I was trying to get at in this part in class yet. But in general, when I ask broad questions like this, I'm looking for you to use your intuition and even imagination, and don't have an exact "right answer" in mind.

The government will probably want to take a broader social perspective on costs. Specifically, it might want to take account of the environmental costs due to the electricity generation needed to power the lights in the building. This will make the fluorescent bulbs look better still, because they require less electricity. The government will probably want to use a discount rate other than the market interest rate as well. If the government is concerned about not placing less ethical weight on the welfare of future generations that will use the building compared to the welfare of the present generation, it will want to use a social discount rate, maybe as low as 1%. If the government is more concerned about the costs of removing funds from private use through taxation of the present generation in order to fund the building, it will want to use a higher discount rate, perhaps based on the average private before-tax return on investment of 7% or higher if the tax will come primarily from owners of capital. There are lots of other possible answers to this question. Perhaps the government places some value on supporting the incandescent light bulb industry, in which case it may take the incandescent option even against the recommendation from a cost analysis. And so on.

3. (Adapted from *Environmental and Natural Resource Economics: A Contemporary Approach* by Jonathan M. Harris.) The World Bank is considering a proposal from the country of Equatoria for a large dam project. Construction costs will be \$500 million per year in each of "year -2", "year -1" and year 0. Operating costs as of year 1 will be \$50 million per year. In addition, there will be costs associated with lost agricultural land of \$40 million per year and associated with lost forest lands of \$20 million per year, again beginning with dam operation in year 1. There will also be costs associated with displaced villagers, watershed damage and ecosystem destruction, but estimates of the monetary value of

these costs are not available. The benefits as of the beginning of dam operation will be annual production of 3 billion kilowatt hours of electricity (valued at the market price of \$0.05 per kilowatt hour) and 5 billion gallons of water available for irrigation (valued at the market price of \$0.015 per gallon).

a) Suppose that the time horizon of the project is 20 years and that the discount rate to be used for evaluating the project is 12%. In the 21st year, the dam and all of the affected land will be sold to a private oil exploration company for \$9.6 billion. Calculate the net present value of the dam project taking into account all quantifiable costs and benefits. On this basis, should the World Bank approve Equatoria's proposal, reject it or explore it in more detail? (Hint: it will be easiest to bring the construction costs in years -2 and -1 up to present value as of year 0. Just do this by multiplying by the discount factor raised to the appropriate power. Convince yourself that it makes sense that \$500 paid two years ago is more costly than \$500 paid today. This is a simple way to compress all start-up values into year 0, then proceed as we usually would.)

answer:

There is a mechanical way to think of discounting backwards. Remember that if, for example, we receive \$5 in four years, we say that the present value to us today is  $\delta^4 5$ . So if we received \$5 two years ago, the present value to us today is  $\delta^{(-2)} 5$ . To think about this more conceptually, note that, since  $\delta = 1/(1+r)$  and a negative exponent causes fractions to be inverted, this is equivalent to  $5(1+r)^2$ . In other words, it's like saying the \$5 has accumulated interest for two years. That should make conceptual sense: \$5 received two years ago (or \$500 paid out two years ago) should be worth more than if the same amount were received today (or should hurt more than \$500 paid today) because two years ago we could have put that amount in the bank and let it accumulate interest.

Based on this reasoning, let the present value of total construction costs be given by  $C_0 = 500 + 500(1.12) + 500(1.12)^2 = \$1,687.2$  million.

Annual (quantifiable) costs =  $50 + 40 + 20 = \$110$  million

Annual benefits =  $3000 * 0.05 + 5000 * 0.015 = \$225$  million

Annual net benefits =  $225 - 110 = \$115$  million.

$\delta = 1/(1.12) = 0.8929$

$$\begin{aligned} \text{NPV} &= -C_0 + \frac{\delta - \delta^{21}}{1 - \delta} \text{NB} + \delta^{21} B_{21} \\ &= -1687.2 + \frac{0.8929 - 0.8929^{21}}{1 - 0.8929} 115 + 0.8929^{21} 9600 \\ &= -1687.2 + 859.0 + 888.6 \\ &= \$60.4 \text{ million.} \end{aligned}$$

On this basis, it looks like the World Bank should approve this proposal. However, remember that there are costs that we recognize are present but weren't able to include in the NPV calculation because we weren't able to quantify or monetize them (displacement, ecosystem damage, etc.). In situations like this one, it would probably be worthwhile to undertake a contingent valuation survey or otherwise take the time and expense to monetize these costs. The resulting estimates of the costs may be small enough that

the NPV will remain positive, but it is also possible that these costs that have been left out are substantial and would tip the balance between approving and rejecting.

b) Suppose the World Bank eventually rejects the proposal, but suggests the following alternative project: several small-scale dams will be constructed instead of a single large dam. This alternative project will lead to virtually zero costs due to lost agricultural and forest land, as well as to displacement and watershed and ecosystem damage. Electricity and water production will be half as much per year as with the large dam (while their respective prices will remain constant), but annual operating costs will also be halved. Construction will still be spread over years -2, -1 and 0, but costs will be only \$210 million in each of these years for this alternative project. It will no longer be possible to sell the land with this alternative project, so the government would be required to adopt an infinite planning horizon. Still using a discount rate of 12%, calculate the net present value of this alternative project. Does it seem desirable?

answer:

$$PV \text{ of construction costs} = 210 + 210(1.12) + 210(1.12)^2 = \$708.6 \text{ million.}$$

$$\text{Annual costs} = 50/2 = \$25 \text{ million}$$

$$\text{Annual benefits} = 1500 \cdot 0.05 + 2500 \cdot 0.015 = \$112.5 \text{ million}$$

$$\text{Annual net benefits} = 112.5 - 25 = \$87.5 \text{ million.}$$

$$NPV = -C_0 + NB/r$$

$$= -708.6 + 87.5/0.12$$

$$= -708.6 + 729.2$$

$$= \$20.6 \text{ million.}$$

This alternative project seems desirable relative to the status quo due to the positive NPV. On casual observation it may appear undesirable compared to the large-scale project from the previous part (due to its lower NPV), but remember that the large-scale dam would entail unmeasured costs that the small-scale project avoids. Would the large-scale project still have a higher NPV after these additional costs were accounted for? It's impossible to know without somehow estimating those additional costs.

In a scenario like this one, the World Bank's project analysts could either take more time to quantify and monetize the additional costs of a given proposed project (as was suggested in the answer to the first part), or could alter the proposed project so that it does not entail any costs that cannot be included in the NPV calculation (as was just done here). With the latter approach, there is a risk that they end up recommending a project that is less desirable than the alternative (based on an ideal comparison taking account of all costs). But at least they are not at risk of recommending a project that is worse than the status quo, and in addition, they avoid what could be the very large complication and expense of quantifying the additional costs. What they really would *not* want to do is to make a decision that completely ignores the additional costs that the large-scale dam entails (which is what we would implicitly be doing if we just took the 60.4 figure from the first part at face value).