EC371 – Environmental Economics

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Practice Problems for Unit 3: Air Pollution – Solutions

There are three problems. Please read and think about them carefully, and work through them before *looking at the solutions. If you are having trouble, you can seek clarification and help from classmates and during my office hours, but it is highly recommended that you struggle through the questions yourself first. Your goal should be both to learn the mechanics and to grasp the intuition and think more deeply about the issues. Solutions will be posted by the afternoon of Friday, November 11, 2011. If you would like comments on your work and solutions, you can submit them to me at any time.*

1. Consider an economy with two firms that emit an environmentally harmful uniformly mixed fund pollutant as a by-product of their production process. These emissions are perfectly and costlessly monitored by the government. Suppose that it has been decided that there should be a *reduction* in total emissions of 21 units. The marginal cost relations faced by each firm for abating a given amount are $MC_1 = 100q_1$ and $MC_2 = 200q_2$ (in dollars) where q_1 and q_2 are the units of reduction (i.e. abatement) undertaken by firm 1 and firm 2 respectively. Each firm produces 32 units of *emissions* in the absence of any regulation, i.e. each firm has baseline emissions of 32 units. (You should be able to show that the aggregate abatement target of 21 together with aggregate baseline emissions imply that the aggregate emissions target is 43.)

a) Find the cost-effective allocation of individual abatement requirements. Does it make intuitive sense? Calculate the appropriate per-unit fee that the government would have to implement to achieve this allocation. Calculate the amount that would be paid by each firm towards total abatement costs and total fees to the government if this fee were in place.

answer:

Note that this is almost the same as the example we did in class, but not exactly the same.

Apply the equimarginal principle ($MC_1 = MC_2$) and the reduction constraint ($q_1 + q_2 = 21$):

$100q_1 = 200(21 – q_1)$
$q_1^{ce} = 14$, $q_2^{ce} = 21 – 14 = 7$

(makes sense because firm 1 has half the marginal costs as firm 2 for a given reduction and ends up abating twice as much in the cost-effective allocation)

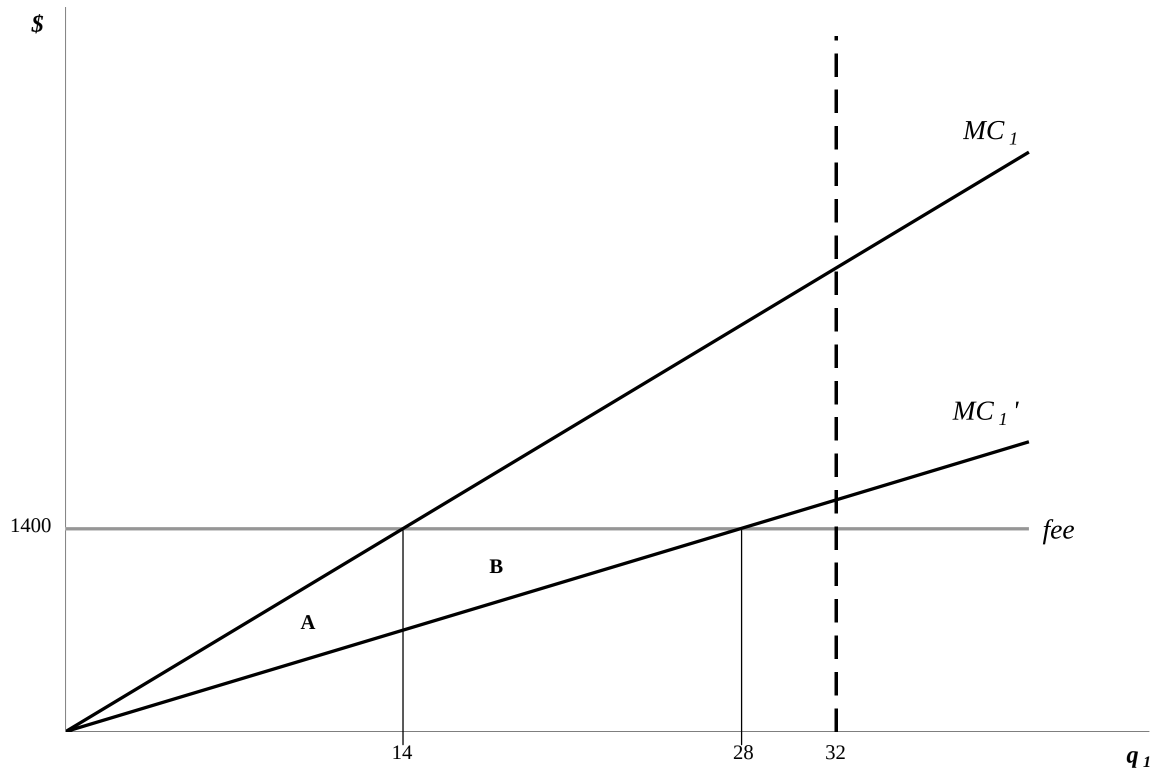
fee = MC at cost-effective allocation = 100(14) = $1,400 per unit (= 200(7))

firm 1 fees: (32 – 14)*1400 = $25,200
firm 2 fees: (32 – 7)*1400 = $35,000
firm 1 abatement costs = 1/2*14*1400 = $9,800
firm 2 abatement costs = 1/2*7*1400 = $4,900

(you should be able to show these areas on graphs too)

b) Suppose that firm 1 has access to a new technology, which would cause its marginal abatement cost relation to become $MC_1' = 50q_1$ if adopted. Calculate the immediate benefit that the firm would enjoy if it were to adopt the new technology, assuming that the per-unit fee stays constant at the level you found in the previous part. Describe the two sources of this benefit. What are total industry-wide emissions after firm 1 adopts the technology?

answer:



On the graph above, the immediate benefit (gross of the cost of the technology) is given by the area A plus the area B. Area B is a right triangle, and area A can be calculated as the area of a rectangle minus the areas of two right triangles (or other ways).

area A = 1400*14 – 1/2*1400*14 – 1/2*700*14 = $4,900
area B = 1/2*(1400 – 700)*(28 – 14) = $4,900
gross benefit = 4900 + 4900 = $9,800.

(Do these calculations yourself to make sure you see where they are coming from! Of course, you might find it quicker to use calculus, which is fine too. As usual, if you use a slightly different method but get the same answer, that is fine as long as you understand what you are doing.)

Area A corresponds to savings due to paying lower per-unit abatement costs on the units already being abated. Area B corresponds to savings due to re-minimizing costs by setting the new marginal cost curve to the per-unit fee of 1400 and thus paying the lower marginal abatement costs for the next several units relative to the fee that would otherwise have to be paid on these units.

When firm 1 re-optimizes, it will choose 28 units of abatement (by setting $50q_1 = 1400$ to minimize costs). Nothing has changed for firm 2, so it will keep abating 7 units, for a total reduction of 28 + 7 =

35 units, i.e. the aggregate abatement target of 21 will be exceeded. Since total pre-regulation emissions are 64, this corresponds to total (post-regulation) emissions of 64 – 35 = 29.

c) Instead of using an emissions fee, suppose that the government prints 43 tradable emissions permits, gives 12 to firm 1 and the remaining 31 to firm 2, and sets penalties for over-emitting so stringent that neither firm ever wants to emit beyond the number of units that it is allowed to as determined by the number of permits that it holds. Firm 1 initially has the old technology (i.e. $MC_1 = 100q_1$). What is the cost-effective allocation of individual *emissions*? Describe in some detail, with a few calculations, the process by which this allocation will be achieved starting from the initial allocation of permits (and assuming that there are no frictional or transaction costs that would hinder bargaining between firms). What will be the price of permits *at this allocation* (i.e. at what price would the last permit be exchanged if each permit were for an infinitesimal unit of emissions)?

answer:

We already know that the cost-effective allocation of *abatements* is 14 and 7 for firm 1 and firm 2 respectively, so we just need to convert these to emissions: 32 – 14 = 18 and 32 – 7 = 25 respectively.

At the initial allocation of emissions [12,31], firm 1 has too few permits and firm 2 has too many compared to the cost-effective allocation [18,25]. So the movement from the initial to the cost-effective allocation will involve a transfer of permits from firm 2 to firm 1 – six permits, to be precise, if each is for a discrete unit of emissions. (If you were to draw this situation on a graph with q_1 on the horizontal axis – which I encourage you to do – the initial allocation would be six units to the *right* of the cost-effective abatement allocation, which of course is itself where the two marginal cost functions intersect.)

Firm 2 can be persuaded to give up a permit if it is compensated for doing so. In this case, firm 2 would be willing to abate a second unit (check to make sure that you understand that firm 2 is abating one unit at the initial allocation) for $400 (plug 2 into firm 2's marginal abatement cost function). Firm 1, on the other hand, can save $2,000 by abating 19 units rather than the 20 units that it is forced to reduce by in the initial allocation. Obviously there is scope for bargaining in terms of firm 1 paying at least $400 and up to $2,000 to acquire a permit from firm 2. For an additional permit, firm 2 will need to be compensated by at least its $600 marginal abatement cost for the additional unit it will need to abate, but firm 1 will be willing to pay up to $1,900 for having to abate only 18 rather than 19 units, so there is clearly still a large bargaining range. This same argument can be repeated until the cost-effective allocation is reached: the price of the last permit will be close to $1,400, where marginal costs of abatement are equalized (or exactly this if trading of infinitesimal units were possible).

d) How much will each firm pay in terms of total abatement costs *at this allocation* plus total permit purchases made in getting to this allocation? (To make things as simple as possible, assume that each permit is for one discrete unit of emissions, and also that each permit is exchanged at the terminal price you found in the previous part.) How does the total cost burden on firms in this case compare to that with the fee policy from part a)?

answer:

The abatement costs incurred by each firm at the cost-effective allocation are the same as those calculated in part a). This must be the case, since the same allocation has been obtained. We do not know exactly what each permit will be sold for because it depends on the relative bargaining power of the firms. For illustration, we can assume that each permit is sold at some price, say $1,400 as I suggested in the question. Firm 1 buys six permits, so pays 6*1400 = $8,400, while firm 2 receives this amount as revenue (and pays nothing for permits, because it only sells permits, and does not buy any). Obviously, total costs to each of the two firms are much lower in this case than with the fee in part a). This is especially the case for firm 2, which, in addition to no longer having to pay fees, actually receives some payments from firm 1 for permits that much more than offset its abatement costs, thus coming out ahead. Since the permit payments wash out between the two firms, for the industry as a whole only abatement costs are borne; the industry no longer has to pay the $60,200 that was previously being collected as fees.

e) Now suppose that, with the permit policy in place and starting from this allocation, firm 1 adopts the new technology (i.e. its marginal cost curve becomes $MC_1' = 50q_1$). Explain briefly and *intuitively* how the allocation of emissions, the aggregate level of emissions and the price of permits will change.

answer:

The new cost-effective allocation (which you do not need to calculate or show graphically in order to answer this part, though that might be good practice) will involve an abatement allocation that is more biased towards firm 1, i.e. it will involve higher abatement by firm 1 than the previous cost-effective allocation did. Why? Because with the new technology in place at firm 1, the industry as a whole can save on abatement costs by relaxing firm 2's reduction target and tightening firm 1's reduction target, i.e. by shifting the abatement burden slightly more towards where abatement is now even cheaper to do. There is a big difference between this situation (with a permit system) and the situation in part b) (with a fee in place): total emissions will stay fixed at 43, because that's how many permits are still available in the aggregate. So if firm 1 is to abate more, firm 2 must be abating less, or in other words, firm 2's emissions will increase moving from the old to the new cost-effective allocation. The permit price will fall. Mechanically, this is because the marginal costs are now equalized at a lower level for the given reduction target. Intuitively, this is because of the reasoning just developed, which holds for the individual firms as much as it does for the industry as a whole. When firm 1 adopts the new technology, its MC curve rotates down, and this opens up a bargaining range over permits: firm 1 is willing to sell permits for a price that's lower than what firm 2 would be willing to pay in order to acquire additional permits and thus lighten its abatement burden. Permits will thus be exchanged – being sold by firm 1 to firm 2 – until the new cost-effective allocation is reached. The bargaining range for prices of these exchanged permits will involve lower prices than was the case in the previous part. Essentially, the right to pollute has become less valuable because it has become easier (i.e. cheaper) to reduce pollution.

2. The marginal damage costs from emissions of a uniformly mixed fund pollutant are given by $MD = 0.09Z^2 - 1.25Z + 8.6$, where Z is the aggregate level of emissions of the pollutant at any point in time. The economy-wide marginal control cost curve is given by $MC = 1.25Q + 2.6$, where Q is the aggregate amount of emissions abated at any point in time. The pollutant in question is emitted by a total of four firms, where *each* firm has baseline (current, pre-regulation) emissions of 30 units.

a) Find the efficient level of aggregate emissions of the pollutant in this economy.

answer:

Remember the general equation relating emissions to abatement: abatement = baseline emissions minus regulated emissions. In class we mostly made use of this in the aggregate in terms of the relationship between an emissions target (which we called N) and a reduction target (R), but it's equally valid to treat emissions and abatement in the abstract as variables to be determined. Current emissions in this case are 120 (4 firms multiplied by 30 units each), so we can write this general equation in terms of the notation of this question: $Q = 120 - Z$.

Efficiency is obtained where overall damage plus control costs are minimized, which is where MD=MC as argued in class.

$MD = MC$
$0.09Z^2 - 1.25Z + 8.6 = 1.25Q + 2.6$
$0.09Z^2 - 1.25Z + 8.6 = 1.25(120 - Z) + 2.6$ (substituting in the expression from above)
$0.09Z^2 - 1.25Z + 1.25Z + 8.6 - (1.25*120) - 2.6 = 0$
$0.09Z^2 - 144 = 0$
$Z^2 = 144/0.09$
$Z^* = \sqrt{1600} = 40$.

b) The four firms have marginal abatement cost curves given by $MC_1 = 3 + 4q_1$, $MC_2 = 4 + 5q_2$, $MC_3 = 6 + 5q_3$ and $MC_4 = 7 + 6q_4$ respectively. Suppose that the government decides to mandate (and can fully enforce) the following firm-level reduction targets: firm 1 will abate 29 units; firm 2 will abate 23 units; firm 3 will abate 15 units; and firm 4 will abate 13 units. Will this policy achieve the efficient level of emissions? Will it be cost effective?

answer:

Under the policy, aggregate *abatement* will be 29 + 23 + 15 + 13 = 80. Since, again, abatement = baseline emissions minus emissions under the policy, rearranging this implies that emissions will be 120 – 80 = 40. This is the efficient level of emissions as found in the previous part. So yes, this policy achieves the efficient level of emissions.

To be cost effective, a policy must minimize the total abatement costs of meeting the given target. How do we check if total abatement costs are minimized? The equimarginal principle gives us a shortcut. This principle tells us that if marginal abatement costs are equalized across firms, cost effectiveness will be achieved. So we just need to see if marginal costs are at the same level for each firm at the prescribed abatement allocation. $MC_1 = 3 + 4q_1 = 3 + 4(29) = 119$; $MC_2 = 4 + 5q_2 = 4 + 5(23) = 119$; $MC_3 = 6 + 5q_3 = 6 + 5(15) = 81$; and $MC_4 = 7 + 6q_4 = 7 + 6(13) = 85$. So the first two are equalized, but that is not enough. All four would need to be at the same level for the equimarginal principle to hold. They are not at the same level, so the policy is not cost effective.

Note that it is NOT necessary to calculate the cost-effective allocation to answer this question. Doing so involves solving a system of four equations – three equations that ensure all four marginal abatement costs are equalized, plus the reduction constraint – in four unknowns, which is far too arduous to do by hand (and doesn't give very nice numbers in this case). Also note that it is NOT necessary to make use of any relationship between the economy-wide marginal control cost curve and the individual firm marginal abatement cost curves. (There is, in fact, a theoretical relationship between them, though you won't be required to know or use it, and actually, the economy-wide marginal control cost curve as given isn't quite theoretically correct, because it's been rounded to give cleaner answers.)

As a picky little point of vocabulary (that I'll never try to trick you on but am just pointing out for your interest), note that even though this policy hits the efficient *target*, it's not efficient in itself. For a policy to be fully efficient, cost effectiveness is actually required. The policy in this part is not fully efficient because the same target could be hit at lower total abatement costs by adjusting the individual abatement requirements.

c) Suppose that instead of mandating reduction targets, the government chooses to levy a uniform fee of $115/unit of the pollutant emitted on each firm. Will this alternative policy achieve the efficient level of emissions? Will it be cost effective?

answer:

When faced with a fee, cost-minimizing firms will choose their abatement levels such that their marginal abatement costs are equal to the fee. $MC_1 = 3 + 4q_1 = 115 \rightarrow q_1 = 28$; $MC_2 = 4 + 5q_2 = 115 \rightarrow q_2 = 22.2$; $MC_3 = 6 + 5q_3 = 115 \rightarrow q_3 = 21.8$; and $MC_4 = 7 + 6q_4 = 115 \rightarrow q_4 = 18$. So the abatement allocation achieved by this policy is (28,22.2,21.8,18), which implies aggregate abatement of 28 + 22.2 + 21.8 + 18 = 90. This is higher than the efficient level of aggregate abatement of 80, i.e. the fee is too high relative to the fee that would achieve the efficient level of emissions.

This policy does result in equalized marginal abatement costs across firms, though (at 115, of course). So the policy is a cost-effective way to achieve 90 units of abatement.

3. Consider an economy with two firms that emit an environmentally harmful uniformly mixed fund pollutant as a by-product of their production process. These emissions are perfectly and costlessly monitored by the government. Suppose that it has been decided that there should be abatement of 30 units starting from the pre-regulation level of aggregate emissions. The marginal cost relations faced by each firm for abating a given amount are $MC_1 = 45 + 2(q_1)^2$ and $MC_2 = 9 + (q_2)^2$ (in dollars) where q_1 and q_2 are the units of abatement undertaken by firm 1 and firm 2 respectively. Baseline emissions, in the absence of any regulation, are 27 for firm 1 and 23 for firm 2.

a) Briefly discuss why the shape of the curves might be reasonable in terms of the abatement options available to firms in the real world.

answer:

The curves are quadratic, so that marginal costs are increasing at an increasing rate (the cost of reducing by one more unit rises more than proportionately as more and more units are abated). While not infinite in this example, the cost of abating the very last unit of emissions is astronomically higher than the marginal costs of the first several units. This shape captures at least three characteristics that seem quite reasonable: starting from zero abatement, there will typically be some easy and low-cost opportunities to clean up the first few units; as more units are abated, more equipment being used more intensively will be required to make additional reductions; and the marginal costs of abating the last few units, and hence achieving total reduction (zero emissions), will be prohibitively high.

(If this helps, it might be more concrete to think about a homeowner trying to cut back the electricity bill. To start with, some old incandescent bulbs could be swapped out for new compact fluorescents, which are getting very cheap by now. I would put this firmly in the “low-hanging fruit” category. Then a bit of effort could be expended in breaking some habits, e.g. learning to shut lights off when leaving rooms and plugging electronics that draw electricity in standby mode into power bars that can be switched off when leaving the house for the day. Then additional reductions get more serious – maybe requiring the purchase of a personal-size solar panel. Then, more solar panels and more sophisticated batteries. And finally – especially if the house is in Boston, where you can seemingly go a month without seeing the sun at times – cutting down to zero with all the stored solar energy already used up might require something ludicrous like eating uncooked food by candlelight and going to bed shortly after sundown. Obviously, one would really have to want to avoid paying for electricity from the grid – maybe because the price explodes or something – to be induced to move all the way along this function. An example of a technological paradigm shift that would give rise to a lower marginal abatement cost function in this case might be the emergence of a new generation of more effective solar panels.)

But, having said all this, the first problem should convince you that linear marginal cost schedules can go a long way in illustrating and developing general insights, with the added benefit of keeping the math more manageable. The next part demonstrates the disadvantage of being more realistic: though not complicated math per se, even the baseline questions of interest require a bit more manipulation to answer.

b) Calculate the cost-effective allocation of individual abatement requirements that satisfies the total reduction requirement. If this allocation were to be achieved through implementing an emissions fee, what would be the appropriate amount of the per-unit fee? With this fee in place, how much would each firm remit to the government in the form of fee payments?

answer:

According to the equimarginal principal, cost effectiveness will be achieved, as usual, where

$MC_1 = MC_2$

$45 + 2(q_1)^2 = 9 + (30 - q_1)^2$ (using the reduction constraint)

$(q_1)^2 + 60q_1 - 864 = 0$

$q_1^{ce} = 12$ (discard the negative root because it doesn't make sense)

$q_2^{ce} = 30 - q_1^{ce} = 18.$

Recall that for any given fee f, firms will minimize total costs by abating up to the point that $MC = f$. So to arrive at the cost effective allocation, find the marginal costs associated with the allocation (which will of course be equal for each firm) and set the fee to this level. In this case, $MC_1 = 45 + 2(12)^2 = \$333$ per unit $= f^{ce}$. Total fees remitted will be equal to this fee times the total units *emitted* by the respective firm. Firm 1 was emitting 27 units so will be emitting (27 – 12 =) 15 units with the fee, while firm 2 was emitting 23 units so will be emitting (23 – 18 =) 5 units with the fee. Therefore, total fees remitted are 15*333 = $4,995 and 5*333 = $1,665 respectively.

c) Describe some reasons why the fee policy might be preferred or not to a tradable permits system with permits initially grandfathered, by the firms on the one hand and by the government and society more broadly on the other hand.

answer:

This question is a bit more vague than I usually ask on these sets of practice problems. The goal was to get you thinking about some of the challenges we face in enacting environmental legislation in general. I'll give a few thoughts here, but the point is for you to get some practice with big-picture thinking and supporting your arguments, as opposed to striving for some objectively "right" answer, which isn't really intended to exist for this question.

That being said, I don't think the firm side of things is too cloudy. Firms generally care most about keeping costs low and profits high, and about operating within a framework that allows them to achieve these things consistently over time. A permit system with some initial free distribution of permits entails unambiguously lower private costs for firms than an emissions fee obtaining the same aggregate target would. Further, using a fee would most likely entail some policy iteration, which would be more difficult for firms to make future plans around than would a projected price for acquiring any additional permits as necessary. For firms to potentially prefer fees, we'd have to add more structure to the problem, such as by examining other motives (maybe supporting a fee system is thought to be good public relations or in accordance with perceived notions of corporate social responsibility) or by specifying what the government might do with the fee revenue (perhaps lowering corporate and payroll taxes or subsidizing "clean" investment to partially offset the cost burdens to firms). It may be possible that firms with low baseline emissions would consider grandfathering as giving large entrenched firms an unfair advantage, but while the alternative fee system might mitigate this, it's not clear that small firms would be better on net because of the fee payments they would then be required to make. (There may be other effects related to inequality across firm costs, though, as discussed below.)

For society, the most important outcome of any environmental policy is to achieve the goal at the lowest possible cost in terms of economy-wide real resources, which both instruments accomplish. Whether individual consumers/voters realize it or not, costs imposed on one sector of the economy will ripple through the whole economy, potentially affecting wages, prices, the cost of credit, emergence of new and improved products and so on. From this perspective, what's good for firms is good for society as a whole. The grandfathered permits system not only minimizes real resource costs (the total abatement costs across firms – how we define cost effectiveness), but also entails a lower additional cost of compliance on the part of firms than would a fee system. (The fee revenue is just a transfer from the pockets of firms to the government's pocket, although some could be destroyed in the process of the redistribution by distorting other aspects of firms' behavior. In principle, the government could use this revenue to partially offset the resulting knock-on effects of regulation that negatively affect consumers, but this would require being able to predict and respond quickly to such effects.)

However, society generally cares about a wide variety of things that are obscured by the focus on costs. One of these is equity. There are two equity issues at play here. First, the cost burden on the industrial sector is shared unequally across firms. (In the previous part, we just calculated fee payments. We'd also want to calculate total abatement costs to verify that the total comprehensive cost burden is unequal. With the quadratic shape, this requires integration. We'd probably also want to express the total cost burden for a firm relative to its overall revenues or something else, to make inter-firm comparisons meaningful.) The numbers for this problem were chosen to broadly reflect the stereotype of an industry with some big incumbent firms for whom abatement is expensive, and some small recent entrants for whom abatement is cheaper. Some inequality in cost burdens is inevitable because of the heterogeneity in abatement costs across firms that are at the very heart of the matter, but it can still be useful to look at its sources carefully. In this case, the fee policy looks like it will hurt the big incumbents more than the smaller firms, which might give the incumbents a sharper incentive to innovate in order to stay competitive, reducing the resource cost of abatement effort in the long run. With permits, on the other hand, if the initial allocation is biased enough towards the incumbents, the policy will put smaller firms at a cost disadvantage. This may or may not be something that society cares about directly, but in either case, it could effectively act as a barrier to entry into the industry over time or to dull the incentives for incumbent firms to innovate so as to retain their advantage, both causing the long-term cost of the regulation to be more expensive than otherwise.

The other equity issue is due to the fact that some consumers might be affected more dramatically than others as the effects of the regulation ripple through the economy. If the prices of necessities increase, people with low income will suffer more than others, as they spend a greater proportion of their income on necessities. Workers in some industries might see their wages increase, while the opposite might occur in others. It's not clear specifically how permits and fees might qualitatively differ in effects such as these that they induce, but we can at least say that fees have the advantage of providing the government with revenue that it could possibly use to attempt to counteract such effects.

There are other possible differential impacts of the instruments that we know less about. For example, one or the other could minimize expected efficiency losses under various forms of uncertainty. Instrument choice might also affect the directions in which abatement technology progresses over the long run. A planned permit system might perversely give firms an incentive to increase their emissions leading up to the legislation taking effect so as to ensure a large initial receipt of permits. And so on.

Finally, I intended for you to think about the government and society together, under the implicit assumption that the government is just an agent that acts in society's interests. This need not be the case. Governments are made up of individual politicians, each of whom may care partially about society's goals but may care more about personal or local goals (such as re-election, protecting a home-town company etc.). A permit system with grandfathering would give the most leverage for any given politician to seek a local advantage by influencing the initial distribution of permits – but, of course, many politicians will be thinking this way, and there are only so many permits to go around. This kind of thing falls under the general heading of rent-seeking, which economists have studied in various contexts.