EC371 – Environmental Economics

Instructor: Jeremy Smith

Practice Problems for Unit 4: Natural Resources – Solutions

There is one problem. Please read and think about it carefully, and work through all parts before *looking at the solutions. If you are having trouble, you can seek clarification and help from classmates and during my office hours, but it is highly recommended that you struggle through the questions yourself first. Your goal should be both to learn the mechanics and to grasp the intuition and think more deeply about the issues. Solutions will be posted around the afternoon of Wednesday, November 23, 2011. If you would like comments on your work and solutions, you can submit them to me at any time.*

1. Consider a non-renewable and non-recyclable natural resource that has no substitutes. Society only places value on this resource for the periods specified, and there will be no exploration for this resource over time.

a) If marginal benefits to society are represented by the inverse demand function $P_i = 10 – 0.5Q_i$ for each period $i = 0, 1$ (where Q_i is the quantity of the resource that would be extracted/consumed in period i at price P_i per unit) and marginal extraction costs are $4.00 per unit in each period, what would be the efficient extraction of this resource in each period if the resource were abundant?

answer:

$P_i = MC_i$

$10 – 0.5Q_i = 4$

$Q_i = 12$.

So if there were at least (12 + 12 =) 24 units of the resource available, it would be referred to as abundant, and efficient consumption/extraction would be 12 units per period.

b) Assume that the stock of the resource is fixed at 16 units. Calculate present value marginal net benefit functions from the perspective of period 0 for each period using this same demand and cost information and a discount rate of 7% where necessary. Calculate the dynamically efficient allocation of the resource across the two periods. Does it make sense? Calculate the price in each period that would support the dynamically efficient allocation and the associated marginal user cost in each period.

answer:

$PV(MNB_0) = MNB_0 = 10 – 0.5Q_0 – 4 = 6 – 0.5Q_0$

$PV(MNB_1) = 1/(1.07)*(6 – 0.5Q_1)$

$= 1/(1.07)*[6 – 0.5(16 – Q_0)]$ (making use of the scarcity constraint)

For efficiency, $PV(MNB_0) = PV(MNB_1)$

$6 – 0.5Q_0 = 1/(1.07)*[6 – 0.5(16 – Q_0)]$

$Q^{**}_0 = 8.1353$, $Q^{**}_1 = 16 – 8.1353 = 7.8647$.

It makes sense that we would leave less of the resource for the next generation than we consume today because we discount the benefits enjoyed by the next generation, i.e. the planner today cares more about present consumption than consumption in the next period.

$P^{**}_0 = 10 – 0.5(8.1353) = 5.9324$
$P^{**}_1 = 10 – 0.5(7.8647) = 6.0676$
$MUC_0 = P^{**}_0 – MC_0 = 5.9324 – 4 = 1.9324$
$MUC_1 = P^{**}_1 – MC_1 = 6.0676 – 4 = 2.0676$.

You should be able to think about all of these things graphically too!

c) This type of example can easily be altered to account for i) higher demand in the second period due, for example, to population growth and ii) higher marginal extraction costs in the second period due to the technological complications of extracting the last units of a resource relative to earlier units (e.g. digging deeper and in thinner seams for coal, removing oil from de-pressurized wells, etc.). Consider a situation in which $P_0 = 8 – 0.4Q_0$, $P_1 = 10 – 0.3Q_1$, $MC_0 = 2$, $MC_1 = 3$ and there are 25 units of the resource available. Calculate the dynamically efficient consumption of the resource in the first period, first with a discount rate of 5% and second with a discount rate of 10%. Does the change in the efficient level of first-period consumption with the different discount rate make sense?

answer:

efficient consumption at $r = 5\%$:
$6 – 0.4Q_0 = 1/(1.05)*[7 – 0.3*(25 – Q_0)]$
$Q^{**}_0 = 9.4444$.

efficient consumption at $r = 10\%$:
$6 – 0.4Q_0 = 1/(1.1)*[7 – 0.3*(25 – Q_0)]$
$Q^{**}_0 = 9.595$.

It makes sense that consumption is slightly more skewed to the first period with a 10% discount rate than with a 5% discount rate because we care less about the second period when the discount rate is higher.

(The question did not ask about this, but it is no longer necessarily the case that it would make sense for the efficient allocation to involve greater consumption in the first than in the second period with a given discount rate. If you do the calculations, you will see that efficient consumption in the second period is greater than that in the first with both discount rates. Even though we care less about consumption next period because it is discounted, marginal net benefits are higher in the second period than in the first in this case, and this effect more than offsets the effect from discounting and, for that matter, the effect from the higher marginal cost in the second period.)

d) This type of example can also be easily altered to account for a longer time horizon, although the math becomes more tedious. Recall, however, that the relationship between the marginal user costs across periods associated with the efficient allocation (i.e. the Hotelling Rule) is the same in the many-period case as it is in the two-period case. Suppose that the inverse demand function is $P_i = 12 – 0.3Q_i$ and marginal extraction costs are \$3.00 per unit in each period, and that the relevant discount rate is 6%. Now, however, suppose that the planning horizon is much longer and that the resource is scarce in the sense that, while there are several hundred units available, the market equilibrium would lead to exhaustion of the resource before the end of the planning horizon. Assume that policymakers have calculated and implemented the efficient plan of extraction of the resource, and that there will never be a deviation from this plan. You observe that the price of the resource in period 75 is \$5.6474. Calculate the marginal user cost in period 75 and briefly interpret what it means. What was the efficient level of consumption of this resource for period 74?

answer:

$MUC_{75} = P^{**}_{75} - MC_{75} = 5.6474 - 3 = \$2.6474.$

Conceptually, the marginal user cost in period 75 is the cost (due ultimately to the scarcity of the resource) that the consumption of one extra unit in period 75 would impose on all other periods in the planning horizon.

In words, the Hotelling Rule states that the marginal user cost associated with the efficient extraction plan will rise over time at a growth rate equal to the discount rate. This implies that

$MUC_{75} = (1.06)*MUC_{74}$ or $MUC_{74} = 1/(1.06)*MUC_{75} \rightarrow MUC_{74} = 1/(1.06)*2.6474 = \$2.4975.$

The marginal user cost is by definition the price associated with efficient consumption in a given period minus the associated marginal cost of extraction in the same period. So we just need to add the marginal extraction cost to this to get the price, then use the demand function to go from there to the efficient consumption in period 74.

$MUC_{74} = P^{**}_{74} - MC_{74} \rightarrow P^{**}_{74} = MUC_{74} + MC_{74} = 2.4975 + 3 = \$5.4975.$

$P_{74} = 12 - 0.3Q_{74} \rightarrow Q^{**}_{74} = (12 - P^{**}_{74})/0.3 = (12 - 5.4975)/0.3 = 21.675.$

(I used a planning horizon of 95 periods with 2352 units of the resource available. I did not want to give this information in the question because it is not necessary for the above calculations, and I did not want you to spend time trying to calculate the efficient consumption plan. If you are handy with difference equations, I can show you how the calculation of the efficient consumption plan can be done analytically; and either way, you can apply the steps above to all periods using a spreadsheet if you want to see the time-paths of these variables numerically.)