EC371 – Environmental Economics

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Practice Problem: Renewable Resources – Solutions

The relationship between the growth of a given fish population and the population size can be expressed as $g = 800x - 20x^2$, where g is the addition to the stock in number of fish and x is the size of the stock in thousands of fish.

a) What are the biological equilibria of this fishery? Describe what would happen if, starting from the stable equilibrium population, there were a one-time shock that caused the population to fall by 1,000 fish; and what would happen if, starting from the stable equilibrium population, there were a one-time shock that caused the population to increase by 1,000 fish. Provide calculations of the population size in the first couple of periods following each shock to support your description, but you do not have to continue the calculations until the equilibrium is exactly re-obtained.

answer:

The two biological equilibria are, as discussed in class, where $g = 0$, i.e. $x = 0$ and where $800x = 20x^2$, or $x = 40$. Also as discussed in class, only the latter is stable, and that population level is called the carrying capacity.

To demonstrate numerically what would happen if the fish population suddenly fell from 40 thousand to 39 thousand, we need to make a series of calculations. The growth of the stock associated with the new population level is $g = 800(39) - 20(39)^2 = 780$ fish. So the period after the shock, the new population will be $x_2 = x_1 + g_1/1000 = 39 + 780/1000 = 39.78$ thousand fish. This new population will be associated with a new growth, and so on. By applying these equations sequentially, you will see that the population is climbing back to 40. You can do similar calculations starting at a population level of 41 to see that the population slowly falls back to 40. And of course, in either case, once 40 is reached, growth is zero, so the population will stay there (barring another "shock"). Here are some calculations for the first nine periods following the shock using a spreadsheet:

1	39	41
2	39.78	40.18
3	39.95503	40.03535
4	39.99097	40.00705
5	39.99819	40.00141
6	39.99964	40.00028
7	39.99993	40.00006
8	39.99999	40.00001
9	40	40
10	40	40

b) Suppose that harvesting of this species is 6,000 fish per period. Determine the associated "bionomic" equilibria. Briefly explain what will happen if this harvesting program is started when the fishery is at a population of 35, 25, 15 and 5 thousand fish. No calculations are necessary to support these brief explanations. (Unless you are handy with factoring, you will need to use the quadratic formula to calculate the bionomic equilibria. This is a useful formula to remember.)

answer:

$g - h = 0$

$(800x - 20x^2) - 6000 = 0$

$x = 10$ and 30 thousand fish (using $x = (-800 \pm \sqrt{800^2 - 4*(-20)*(-6000)})/(2*(-20)))$

The second of these will be a stable bionomic equilibrium with sustained harvest of 6,000 fish per period.

Starting from a population of 35,000, harvesting will gradually reduce the population to 30,000, as harvesting is outpacing natural regeneration in each period until the population reaches this level. Starting from population levels of 25,000 and 15,000, the stable bionomic equilibrium population of 30 thousand will again be gradually approached, but in both of these cases this is because we start below the equilibrium population in a region where natural growth is greater than the harvest. But starting from a population of 5,000, we are below the unstable bionomic equilibrium where harvesting will be outpacing regeneration, and the population will be drawn down to zero. (To check these, you can calculate net growth $g - h$ at the given starting population, add this to the starting population and so on, as in the previous part. This is shown in the spreadsheet calculations below.)

1	35	25	15	5
2	32.5	26.5	16.5	2.5
3	31.375	27.655	18.255	0
4	30.78719	28.48302	20.1941	0
5	30.45992	29.04379	22.19335	0
6	30.27172	29.40799	24.09713	0

c) **[NOT REQUIRED]** Find the maximum sustainable yield for this fishery, and the associated population size. (Hint: to find the highest point on this function, you first need to take the derivative of the function and set it to zero. The derivative of the function ax^n with respect to x is nax^{n-1}.)

answer:

By taking the derivative of $g = 800x - 20x^2$ with respect to x and setting it to zero, we will be able to solve for x^*, the population level corresponding to the maximum sustainable yield (and to the highest point on the graph of the g function).

$g' = 800 - 40x = 0 \rightarrow 40x = 800 \rightarrow x^* = 20$ thousand fish.

The maximum sustainable yield will be the number of fish by which the population is growing with population x^*.

$MSY = g^* = 800x^* - 20(x^*)^2 = 800*20 - 20*400 = 8{,}000$ fish.